Name	Date started	Target end date

GCE A LEVEL — FURTHER PURE MATHEMATICS B QUESTION PACK

0978-01 (Legacy FP2) & 0979-01 (Legacy FP3) · New spec A2 Unit 4 Topic 5

REVISE
.wales

FURTHER MATHS — FP B · FURTHER CALCULUS

Further Calculus — Improper Integrals, Volumes & Inverse-Trig Integration

Every further-calculus question from the legacy WJEC FP2 and FP3 papers covering inverse-trig substitution integrals, volumes of revolution and surface area — mapping onto new-spec A2 Unit 4 Topic 5.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 6 minutes

Derived from the legacy FP2/FP3 paper's pace of ***~1.5 min/mark*** *(84 marks over 12 questions). The full Unit 4 exam is* ***2 hours 30 minutes for 120 marks*** *(35% of the A-level qualification).*

You are advised to ***not*** *attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every further calculus question from the legacy WJEC FP2/FP3 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 5 (2.4.5). Unit 4 (Further Pure Mathematics B) is the **120-mark compulsory A2 paper**, worth 35% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 07 Q1	6	
2	Jun 08 Q3	9	
3	Jun 09 Q2	7	
4	Jun 10 Q1	5	
5	Jun 12 Q2	6	
6	Jun 13 Q1	5	

Q	Source	Max	Mark
7	Jun 14 Q2	5	
8	Jun 15 Q1	13	
9	Jun 16 Q1	6	
10	Jun 17 Q2	5	
11	Jun 17 Q3	9	
12	Spec. Q3	8	
	Total	84	

Further Calculus — Improper Integrals, Volumes & Inverse-Trig Integration — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 4: Further Pure Mathematics B · Topic 2.4.5.

Improper integrals 2.4.5

- Integrand undefined at $c \in [a,b]$: $\int_a^b f = \lim_{\varepsilon\to 0^+}\left(\int_a^{c-\varepsilon} f + \int_{c+\varepsilon}^b f\right)$.
- Infinite limit: $\int_a^\infty f\,dx = \lim_{R\to\infty}\int_a^R f\,dx$.
- Convergent iff the limit exists and is finite.
- Common examples: $\int_0^1 dx/\sqrt{x}$ converges; $\int_1^\infty dx/x$ diverges.

Volumes of revolution 2.4.5

- Rotation about x-axis: $V = \pi \displaystyle\int_a^b y^2\,dx$.
- Rotation about y-axis: $V = \pi \displaystyle\int_c^d x^2\,dy$.
- Surface of revolution about x-axis: $S = 2\pi \displaystyle\int_a^b y\sqrt{1+(dy/dx)^2}\,dx$.
- For parametric $x(t), y(t)$: replace dx with $\dot{x}\,dt$ and use $\sqrt{\dot{x}^2+\dot{y}^2}$.

Mean value & inverse trig 2.4.5

- Mean value of f on $[a,b]$: $\bar{f} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.
- $\dfrac{d}{dx}\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$, $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$.
- $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$.
- $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$.

Working scientifically general

- For $1/(a^2-x^2)$: factor and use partial fractions, or trigonometric substitution.
- For $1/(a^2+x^2)$: try $x = a\tan u$ to convert to $\sec^2/\sec^2 = 1$.
- Volumes: always sketch the region first; identify whether it's about x or y axis.
- Improper at both ends: split into two improper integrals at a finite point.

Further Calculus in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Improper integrals — types

Infinite range: $\int_a^\infty f\,dx = \lim_{R\to\infty}\int_a^R f\,dx.$

Unbounded integrand at c: $\int_a^b f\,dx = \lim_{\varepsilon\to 0^+}\int_a^{c-\varepsilon} f\,dx + \int_{c+\varepsilon}^b f\,dx.$

Volumes of revolution

About x-axis: $V = \pi\int_a^b y^2\,dx.$

About y-axis: $V = \pi\int_c^d x^2\,dy.$

For implicit / parametric curves: substitute appropriately.

Surface area of revolution

About x-axis: $S = 2\pi\int_a^b y\sqrt{1+(dy/dx)^2}\,dx.$

About y-axis: $S = 2\pi\int_c^d x\sqrt{1+(dx/dy)^2}\,dy.$

For parametric $(x(t), y(t))$: $\sqrt{\dot{x}^2+\dot{y}^2}\,dt$ replaces the arc-length element.

Mean value

Mean value of f over $[a, b]$:

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Geometric interpretation: height of the rectangle with the same area as $\int f$.

Inverse-trig derivatives

$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$ — $|x| < 1.$

$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$ — $|x| < 1.$

$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$ — all $x.$

Standard inverse-trig integrals

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C.$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C.$

For $1/(a^2-x^2)$ try partial fractions; for $1/(a^2+x^2)$ try $x = a\tan u.$

Trigonometric substitutions

For $\sqrt{a^2-x^2}$ — let $x = a\sin u$, $dx = a\cos u\,du.$

For $\sqrt{a^2+x^2}$ — let $x = a\tan u$, $dx = a\sec^2 u\,du.$

For $\sqrt{x^2-a^2}$ — let $x = a\sec u$ (rarely on FP B; usually use $\cosh u$).

Common pitfalls

- Forgetting to take the limit for improper integrals — just "plugging in ∞" loses marks.
- Sign confusion in volume vs area — V uses y^2, S uses $y\sqrt{1+y'^2}$.
- Forgetting the π in volume formula.
- Wrong substitution: $x = a\sin u$ for $\sqrt{a^2+x^2}$ doesn't simplify.

Strategy

1. Identify the integrand structure (rational, $\sqrt{a^2 \pm x^2}$, etc.).

2. Choose substitution: $\sin$, $\tan$ for $a^2 \pm x^2$; partial fractions for rational.

3. Compute under the substitution; back-substitute or evaluate.

4. For volumes: sketch the region; identify axis; use $\pi\int y^2$ or $\pi\int x^2$.

SECTION T5

Further Calculus

Questions 1–12 · 84 marks

1. Use the substitution $x = y^2$ to evaluate the integral

$$\int_1^4 \frac{\mathrm{d}x}{\sqrt{x(9-x)}},$$

giving your answer correct to two significant figures. [6]

3. *(a)* Using the substitution $u = x^2$, evaluate the integral

$$\int_0^{\sqrt{3}} \frac{x\mathrm{d}x}{(9+x^4)},$$

giving your answer in the form $\frac{\pi}{k}$, where k is an integer. [5]

(b) Evaluate the integral

$$\int_0^1 \frac{\mathrm{d}x}{\sqrt{25-9x^2}}.$$ [4]

2. Using the substitution $u = \tan x$, evaluate the integral

$$\int_0^{\frac{\pi}{6}} \frac{\sec^2 x}{\sqrt{3 - \sec^2 x}} \mathrm{d}x \quad .$$

Explain briefly why the integral could not be evaluated if the upper limit were changed to $\frac{\pi}{3}$. [7]

1. Using the substitution $u = x\sqrt{x}$, evaluate the integral

$$\int_0^2 \frac{\sqrt{x}}{\sqrt{9 - x^3}}\,\mathrm{d}x.$$

Give your answer correct to three decimal places. [5]

2. Using the substitution $u = e^x$, evaluate the integral

$$\int_0^1 \frac{1}{(e^x + 4e^{-x})} dx.$$

Give your answer correct to three decimal places. [6]

1. Using the substitution $u = x^2$, evaluate the integral

$$\int_1^2 \frac{x}{\sqrt{25 - x^4}}\,dx.$$

Give your answer correct to three significant figures. [5]

2. Using the substitution $u = \sin^2 x$, evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{4 - \sin^4 x}}\, dx.$$

Give your answer in the form $\frac{\pi}{k}$, where k is a positive integer. [5]

1. *(a)* Express

$$\frac{5}{(x^2+1)(2-x)}$$

in partial fractions. [4]

(b) Using the substitution $u = \tan x$ and the result in *(a)*, evaluate the integral

$$\int_0^{\frac{\pi}{4}} \frac{5}{2-\tan x}\,\mathrm{d}x.$$

Give your answer correct to three significant figures. [9]

1. Using the substitution $u = x^2$, evaluate the integral

$$\int_0^{\sqrt{2}} \frac{x}{\sqrt{16 - x^4}}\, \mathrm{d}x\,,$$

giving your answer in the form $\dfrac{\pi}{n}$, where n is a positive integer. [6]

2. Evaluate the integral

$$\int_0^2 \frac{2x^2+5}{x^2+4}\,\mathrm{d}x\,,$$

giving your answer in the form $a + b\pi$, where a, b are constants to be determined. [5]

3. The curve C has equation $y = x^3$. The arc joining the points (0, 0) and (1, 1) on C is rotated through an angle 2π about the x-axis. Calculate the curved surface area of the solid generated, giving your answer correct to three significant figures. [9]

3. The arc joining the points (0,0) and (1,1) on the curve $y = x^3$ is rotated through four right-angles about the x-axis.

(a) (i) Show that the area of the curved surface generated is given by

$$2\pi\int_0^1 x^3\sqrt{1+9x^4}\,dx.$$ [2]

(ii) Use the substitution $u = 1 + 9x^4$ to show this area is equal to

$$\frac{\pi}{27}\left(10\sqrt{10}-1\right).$$ [6]

END OF FURTHER CALCULUS PACK

Source: WJEC FP2/FP3 (2008 modular spec) · 2005–2017

Curated for WJEC FM 2017 spec A2 Unit 4 — Topic 5 (2.4.5)