Name	Date started	Target end date

GCE A LEVEL — FURTHER PURE MATHEMATICS B QUESTION PACK

0978-01 (Legacy FP2) & 0979-01 (Legacy FP3) · New spec A2 Unit 4 Topic 4

REVISE
.wales

FURTHER MATHS — FP B · FURTHER ALGEBRA & FUNCTIONS

Further Algebra & Functions — Maclaurin Series & Partial Fractions

Every Maclaurin-series / partial-fractions question from the legacy WJEC FP2 (partial fractions) and FP3 (Maclaurin) papers that maps onto the new-spec A2 Unit 4 Topic 4.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~4 hours 6 minutes

*Derived from the legacy FP2/FP3 paper's pace of **~1.5 min/mark** (164 marks over 15 questions). The full Unit 4 exam is **2 hours 30 minutes for 120 marks** (35% of the A-level qualification).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every further algebra & functions question from the legacy WJEC FP2/FP3 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 4 (2.4.4). Unit 4 (Further Pure Mathematics B) is the **120-mark compulsory A2 paper**, worth 35% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 06 Q7	10	
2	Jun 07 Q3	9	
3	Jun 09 Q5	9	
4	Jun 10 Q4	10	
5	Jun 12 Q4	9	
6	Jun 13 Q6	10	
7	Jun 17 Q6	9	
8	Jun 06 Q3	13	

Q	Source	Max	Mark
9	Jun 07 Q6	14	
10	Jun 09 Q2	9	
11	Jun 10 Q5	13	
12	Jun 11 Q4	11	
13	Jun 13 Q3	9	
14	Jun 17 Q4	16	
15	Spec. Q5	13	
	Total	164	

Further Algebra & Functions — Maclaurin Series & Partial Fractions — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 4: Further Pure Mathematics B · Topic 2.4.4.

Maclaurin series 2.4.4

- For a function f infinitely differentiable at 0: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$.
- General term: $\frac{f^{(n)}(0)}{n!} x^n$.
- Standard series (all valid for stated x):
- $e^x = 1 + x + \frac{x^2}{2!} + \cdots$ — all x. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$ — $|x| < 1$.

More standard series 2.4.4

- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ — all x.
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ — all x.
- $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \cdots$ — $|x| < 1$ unless n is a non-negative integer.
- Combine via algebra (multiply, substitute, differentiate, integrate term-by-term).

Partial fractions 2.4.4

- Denominator $(ax+b)(cx^2+d)$: $\frac{f(x)}{(ax+b)(cx^2+d)} = \frac{A}{ax+b} + \frac{Bx+C}{cx^2+d}$.
- For $(ax+b)(cx+d)(ex+f)$: three linear partial fractions.
- Match coefficients by clearing denominators and equating powers of x, or substitute strategic values.
- Useful for both integration and series expansion.

Working scientifically general

- For Maclaurin of $\ln(1+\sin x)$: substitute $\sin x$-series into $\ln(1+u)$.
- Always state how many non-zero terms are required, then keep one more for safety.
- For composite $f(g(x))$: use the chain rule on f'' or the recurrence f'' involves f, f'.
- Once you have the series, check by plugging $x = 0$ — the constant term must match $f(0)$.

Further Algebra & Functions in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Maclaurin series

For f infinitely differentiable at 0:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$

Standard expansions

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ (all x).

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ (all x).

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ (all x).

More expansions

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots (|x| < 1)$.

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$ ($|x| < 1$ unless $n \in \mathbb{Z}_{\geq 0}$).

$\sinh x, \cosh x$: use $e^x \pm e^{-x}$ form — combine the two series.

Recurrence approach

If f satisfies a differential identity (e.g. $(1+x^2)f''(x) + xf'(x) + f(x) = 0$):

Differentiate n times via Leibniz; evaluate at $x = 0$ to find $f^{(n+2)}(0)$ in terms of $f^{(n+1)}(0), f^{(n)}(0)$.

Composite series

Maclaurin of $\ln(1 + \sin x)$: substitute the $\sin x$-series into $\ln(1+u)$.

Keep only terms up to the requested order; don't propagate higher powers.

Algebraic discipline: multiply out carefully, collect like powers.

Partial fractions: types

Linear factor $(ax+b)$: term $\dfrac{A}{ax+b}$.

Repeated linear $(ax+b)^2$: terms $\dfrac{A}{ax+b} + \dfrac{B}{(ax+b)^2}$.

Quadratic (cx^2+d) irreducible: term $\dfrac{Bx+C}{cx^2+d}$.

Finding coefficients

Method 1: Clear denominators, equate coefficients of like powers of x.

Method 2: Substitute strategic values (roots of factors) to pick off coefficients one at a time.

Cross-check by substituting $x = 0$ or $x = 1$.

Common pitfalls

- Forgetting the range of validity for $\ln(1+x)$ and $(1+x)^n$ series.
- Missing higher-order terms when substituting one series into another.
- Wrong form for irreducible quadratic: needs $Bx + C$ on top, not just B.
- Sign error in $\ln(1+x)$ alternating series.

Strategy

1. For Maclaurin: compute $f(0), f''(0), f'''(0), \ldots$ directly when possible.
2. If derivatives are messy, find a differential relation and use recurrence.
3. For partial fractions: write the form first, then solve for constants.
4. Always verify by combining the partial fractions back.

SECTION T4

Further Algebra and Functions

Questions 1–15 · 164 marks

7. *(a)* Express

$$\frac{x}{(x+2)(x^2+4)}$$

in partial fractions. [4]

(b) Hence evaluate the integral

$$\int_2^3 \frac{x}{(x+2)(x^2+4)}\,\mathrm{d}x,$$

giving your answer correct to three decimal places. [6]

3. Let

$$f(x) = \frac{(x+1)(x+2)}{(x-1)(x^2+1)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Find $\int f(x)\,\mathrm{d}x$. [4]

5. The function f is defined by

$$f(x) = \frac{1}{(x+1)(x+2)(x+3)} \ .$$

(a) Express $f(x)$ in partial fractions. [4]

(b) Evaluate the integral

$$\int_0^5 f(x)\,\mathrm{d}x \ ,$$

giving your answer in the form $\ln\left(\frac{m}{n}\right)$ where m, n are integers. [5]

4. The function f is defined by

$$f(x) = \frac{3x^2}{(x+2)(x^2+2)}.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) Evaluate the integral

$$\int_1^2 f(x)\,\mathrm{d}x.$$

[6]

4. The function f is given by

$$f(x) = \frac{3x^2 - 4x + 1}{(x-2)(x^2+1)}.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) Hence evaluate

$$\int_3^4 f(x)\mathrm{d}x,$$

giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a, b are positive integers. [5]

6. *(a)* Express

$$\frac{4x^2 - 2x + 9}{x\left(x^2 + 3\right)}$$

in partial fractions. [4]

(b) Hence evaluate

$$\int_1^3 \frac{4x^2 - 2x + 9}{x\left(x^2 + 3\right)}\,\mathrm{d}x,$$

giving your answer correct to three significant figures. [6]

6. The function f is defined by

$$f(x)=\frac{24x^2+31x+9}{(x+1)(2x+1)(3x+1)}\,.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) (i) Evaluate the integral

$$\int_0^2 f(x)\,\mathrm{d}x\,,$$

giving your answer as $\ln N$, where N is a positive integer.

(ii) Explain briefly why the integral

$$\int_{-2}^0 f(x)\,\mathrm{d}x$$

cannot be evaluated. [5]

3. The function f is defined by

$$f(x) = \ln \sec x.$$

(a) Find the Maclaurin series of $f(x)$ up to and including the term in x^4. [9]

(b) The equation

$$\ln \sec x = 1 - 10x^2$$

has a small positive root α. Use your series to find an approximation to α, giving your answer correct to four decimal places. [4]

6. The function f is defined by

$$f(x) = \ln \tan\left(\frac{\pi}{4} + x\right).$$

(a) Show that

$$f'(x) = 2\sec 2x.$$ [4]

(b) Find the first two non-zero terms in the Maclaurin expansion of f. [7]

(c) The equation

$$f(x) = 10x^3$$

has a small positive root. Find its approximate value. [3]

2. Find the first three non-zero terms of the Maclaurin series of $\ln(2 - e^x)$. [9]

5. Consider the function

$$f(x) = \ln(1 + \sinh x).$$

(a) (i) Find the first three non-zero terms of the Maclaurin series for $f(x)$.

(ii) Explain how your result enables you to conclude that f is neither an odd function nor an even function. [10]

(b) The equation

$$\ln(1 + \sinh x) = 10x^2$$

has a small positive root. Use your result in *(a)*(i) to find its approximate value, giving your answer correct to two significant figures. [3]

4. The function f is defined by

$$f(x) = e^x \cos x.$$

(a) Show that

$$f''(x) = -2e^x \sin x. \qquad [2]$$

(b) Determine the Maclaurin series for $f(x)$ as far as the term in x^4. [5]

(c) By differentiating your series, determine the Maclaurin series for $e^x \sin x$ as far as the term in x^3. [4]

3. The function f is defined by

$$f(x) = \ln(2e^x - 1).$$

(a) Show that

$$f''(x) = \frac{-2e^x}{(2e^x - 1)^2}.$$

[3]

(b) Determine the Maclaurin series for $f(x)$ as far as the term in x^3. [6]

4. The function f is defined by

$$f(x) = \cos(\ln(1 + x)).$$

(a) Show that

$$(1 + x)^2 f''(x) + (1 + x) f'(x) + f(x) = 0.$$ [4]

(b) Hence, or otherwise, show that the Maclaurin series for $f(x)$ is

$$1 - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \ldots$$ [5]

(c) Deduce the Maclaurin series for $\sin(\ln(1 + x))$ as far as the term in x^2. [4]

[9]

(b) Use your series to evaluate, approximately, the integral

$$\int_0^{\frac{1}{3}} \ln(1+\sin x)\,dx$$ [4]

END OF FURTHER ALGEBRA & FUNCTIONS PACK

Source: WJEC FP2/FP3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 4 — Topic 4 (2.4.4)