Name	Date started	Target end date

GCE A LEVEL — FURTHER PURE MATHEMATICS B QUESTION PACK

0977-01 (Legacy FP1) · New spec A2 Unit 4 Topic 3

REVISE
.wales

FURTHER MATHS — FP B · MATRICES 3 × 3

Matrices — Determinants, Inverses & Three Simultaneous Equations

Every 3×3 matrix question from the legacy WJEC FP1 papers (January 2006 – June 2017 + Specimen) that maps onto the new-spec A2 Unit 4 Topic 3. Note: legacy FP2/FP3 do not cover 3×3 matrix work — the algebraic technique was housed in legacy FP1 under the 2008 modular spec.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~3 hours 10 minutes

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (127 marks over 14 questions). The full Unit 4 exam is **2 hours 30 minutes for 120 marks** (35% of the A-level qualification).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every matrices 3 × 3 question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 3 (2.4.3). Unit 4 (Further Pure Mathematics B) is the **120-mark compulsory A2 paper**, worth 35% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jan 07 Q2	8	
2	Jan 08 Q3	9	
3	Jan 10 Q4	9	
4	Jan 11 Q6	11	
5	Jan 13 Q4	10	
6	Jan 14 Q7	7	
7	Jun 07 Q7	12	
8	Jun 08 Q2	8	
9	Jun 09 Q3	8	
10	Jun 10 Q3	10	
11	Jun 12 Q4	8	
12	Jun 14 Q6	9	
13	Jun 17 Q1	7	
14	Spec. Q8	11	
	Total	**127**	

Matrices — Determinants, Inverses & Three Simultaneous Equations — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 4: Further Pure Mathematics B · Topic 2.4.3.

Determinant of 3×3 2.4.3

- For $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$: $\det A = a(ei - fh) - b(di - fg) + c(dh - eg)$.
- Expansion can be along any row or column (with cofactor sign pattern $+ \; - \; +$).
- $\det A$ is the signed volume scale factor of the linear map A on $\mathbb{R}^3$.
- Sign indicates orientation: $\det A > 0$ preserves, $\det A < 0$ reverses.

Inverse via adjugate 2.4.3

- Cofactor matrix C: entries $C_{ij} = (-1)^{i+j} M_{ij}$ where M_{ij} is the (i, j)-minor.
- Adjugate $\operatorname{adj}(A) = C^T$ (transpose of the cofactor matrix).
- Inverse: $A^{-1} = \dfrac{1}{\det A} \operatorname{adj}(A)$, valid iff $\det A \neq 0$.
- Verification: $AA^{-1} = I$.

Solving simultaneous equations 2.4.3

- Write $A\mathbf{x} = \mathbf{b}$; if $\det A \neq 0$, unique solution $\mathbf{x} = A^{-1}\mathbf{b}$.
- If $\det A = 0$: either no solution (inconsistent) or infinitely many (dependent).
- Reduce to echelon form to distinguish these cases.
- Geometric: three planes — unique intersection (point), line of intersection, or no common point.

Working scientifically general

- Always check $\det A$ first — everything else hinges on it.
- For parameterised matrices (λ in entries), $\det A = 0$ gives the singular values of λ.
- Use echelon form (row reduction) when the matrix is singular — faster than adjugate.
- For A^{-1}, double-check by computing one entry of AA^{-1}.

Matrices 3 × 3 in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Determinant

For $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$:

$$\det A = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$\det A \neq 0 \Leftrightarrow A$ is non-singular and invertible.

Cofactor matrix

M_{ij} = determinant of the 2×2 submatrix obtained by deleting row i and column j.

Cofactor: $C_{ij} = (-1)^{i+j} M_{ij}$.

Sign pattern: $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$.

Adjugate & inverse

$\text{adj}(A) = C^T$ (transpose of the cofactor matrix).

$$A^{-1} = \frac{1}{\det A} \text{adj}(A)$$

Always check: $AA^{-1} = I_3$.

Three simultaneous equations

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

Write as $A\mathbf{x} = \mathbf{b}$; if $\det A \neq 0$, unique $\mathbf{x} = A^{-1}\mathbf{b}$.

Echelon form

Row-reduce the augmented matrix $[A|\mathbf{b}]$ to row-echelon form.

Use row operations: swap rows, scale, add multiple of one row to another.

From echelon form: back-substitute or read off solution / spot inconsistency.

Cases for $\det A = 0$

No solution: equations inconsistent (planes don't share a common point).

Infinitely many: equations dependent (line / plane of solutions).

Distinguish via echelon form: contradictory row $0 = c \neq 0$ means no solution.

Geometric meaning

Each linear equation in x, y, z defines a plane in $\mathbb{R}^3$.

Three planes can: (a) meet at a unique point, (b) meet in a line, (c) be parallel (no common point), (d) coincide (infinitely many).

$\det A$ is the scaled volume of the parallelepiped spanned by the row vectors.

Common pitfalls

- Sign error in the cofactor expansion — use the $+ - +$ checkerboard.
- Forgetting to transpose for adjugate (it's not just the cofactor matrix).
- Dividing by $\det A$ when $\det A = 0$ — the inverse does *not* exist.
- Mis-identifying "no solution" vs "infinitely many" — use echelon form.

Strategy

1. Compute $\det A$ first.
2. If $\det A \neq 0$: compute A^{-1} via adjugate / cofactor.
3. Multiply: $\mathbf{x} = A^{-1}\mathbf{b}$.
4. If $\det A = 0$: row-reduce to determine which case (no sol vs ∞ many).

SECTION T3

Matrices 3x3

Questions 1–14 · 127 marks

2. *(a)* Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

[6]

(b) **Hence** solve the equations

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + 3y + z &= 4 \\ 3x + 4y + 2z &= 4. \end{aligned}$$

[2]

3. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & \lambda & 2 \end{bmatrix} .$$

(a) Find the value of λ for which **A** is singular. [3]

(b) Given that $\lambda = 4$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse of **A**. [6]

4. *(a)* Show that the following matrix is singular.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 4 & 5 & 7 \end{bmatrix}$$ [2]

(b) Consider the following equations

$$\begin{aligned} x + 2y + 2z &= 1 \\ 2x + y + 3z &= 3 \\ 4x + 5y + 7z &= \lambda \end{aligned}$$

(i) Find the value of λ for which these equations are consistent.

(ii) Find the general solution corresponding to this value of λ. [7]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ \lambda & 1 & -2 \\ 2 & 1 & \lambda \end{bmatrix}.$$

(a) (i) Find and simplify an expression for the determinant of **A**.

(ii) Show that **A** is non-singular for all real values of λ. [4]

(b) Given that $\lambda = 1$,

(i) find $\mathbf{A}^{-1}$, the inverse of **A**,

(ii) hence solve the equation **AX** = **B**,

$$\text{where } \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and } \mathbf{B} = \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}.$$ [7]

4. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & 3 & \lambda \\ 4 & 7 & 5 \end{bmatrix}.$$

(a) Find the values of λ for which **A** is singular. [5]

(b) Given that $\lambda = 1$,

(i) determine the adjugate matrix of **A**,
(ii) determine the inverse matrix $\mathbf{A}^{-1}$. [5]

7.

(a) Given that $\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$,

(i) find the adjugate matrix of $\mathbf{A}$,

(ii) find the inverse of $\mathbf{A}$. [5]

(b) **Hence** solve the equations

$$\begin{aligned} 2x + 3y + z &= 13, \\ x + 2y + 3z &= 13, \\ 2x + 3y + 4z &= 19. \end{aligned}$$

[2]

7. *(a)* Show that the matrix **A** defined below is singular.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 4 & 1 \\ 1 & 8 & -5 \end{bmatrix}$$ [3]

(b) (i) Find the value of k for which the following equations are consistent.

$$\begin{aligned} 2x + y + 2z &= 3 \\ 3x + 4y + z &= 1 \\ x + 8y - 5z &= k \end{aligned}$$

(ii) For this value of k, find the general solution of these equations. [9]

2. *(a)* Find the inverse of the matrix

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

[6]

(b) **Hence** solve the equations

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 5 \end{bmatrix}.$$

[2]

3. *(a)* Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix}.$$ [6]

(b) Hence solve the system of equations

$$\begin{aligned} x + 2y + 3z &= 13 \\ 2x + 3y + z &= 13 \\ 3x + 5y + 2z &= 22. \end{aligned}$$ [2]

3. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 2 & \lambda & 3 \\ 1 & 2 & \lambda \\ 4 & 5 & 5 \end{bmatrix}$$

(a) Find the values of λ for which **A** is singular. [4]

(b) Given that $\lambda = 3$,

(i) find the inverse of **A**,

(ii) **hence** solve the equations

$$\begin{aligned} 2x + 3y + 3z &= 2 \\ x + 2y + 3z &= -1 \\ 4x + 5y + 5z &= 4. \end{aligned}$$

[6]

4. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 1 & 4 \\ 4 & 5 & 7 \end{bmatrix}.$$

(a) (i) Find the adjugate matrix of **A**.
(ii) Find the inverse of **A**. [6]

(b) **Hence** solve the equations

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 1 & 4 \\ 4 & 5 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix}.$$ [2]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 2 & 3 \\ -1 & 1 & 1 \\ 2 & \lambda & 2 \end{bmatrix}.$$

(a) Find the values of λ for which **A** is singular. [4]

(b) Given that $\lambda = -1$,

(i) find the adjugate matrix of **A**,
(ii) find the inverse of **A**. [5]

1. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}.$$

(a) Evaluate the determinant of $\mathbf{M}$. [2]

(b) (i) Find the adjugate matrix of $\mathbf{M}$.

(ii) Deduce the inverse matrix $\mathbf{M}^{-1}$. [3]

(c) Hence solve the system of equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \\ 17 \end{bmatrix}.$$

[2]

8. (a) Find the value of λ for which the following matrix is singular.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ \lambda & 1 & 5 \end{bmatrix}$$ [3]

(b) (i) Find the value of μ for which the following system of equations is consistent.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ \mu \end{bmatrix}$$

(ii) For this value of μ, find the general solution to this system of equations. [8]

END OF MATRICES 3 × 3 PACK

Source: WJEC FP1 (2008 modular spec) · 2005–2017

Curated for WJEC FM 2017 spec A2 Unit 4 — Topic 3 (2.4.3)