Name | Date started | Target end date

GCE A LEVEL — FURTHER PURE MATHEMATICS B QUESTION PACK

0982-01 (Legacy M3) · New spec A2 Unit 4 Topic 10

REVISE .wales

Differential Equations — 2nd-Order Linear (Auxiliary Equation & Particular Integral)

Every second-order linear differential-equation question from the legacy WJEC M3 papers (2006–2017) that maps onto new-spec A2 Unit 4 Topic 8 (2.4.8 second-order linear DEs with constant coefficients).

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2h 39m

Derived from the legacy M3 paper's pace of ~1.5 min/mark (106 marks over 8 questions). The full Unit 4 exam is 2 hours 30 minutes for 120 marks (35% of the A-level qualification). You are advised to not attempt to complete this in one sitting.

ABOUT THIS QUESTION PACK

This is a comprehensive practice question pack focused on second-order linear DEs with constant coefficients. It contains every 2nd-order DE question from the legacy WJEC M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 8 (2.4.8). Questions cover the auxiliary equation (real distinct, real repeated, and complex roots), the complementary function, and the particular integral for polynomial forcing terms.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

FOR EXAMINER'S USE ONLY

Q	Source	Max	Mark
1	Jun 06 Q2	12	
2	Jun 10 Q3	12	
3	Jun 12 Q3	12	
4	Jun 13 Q3	14	
5	Jun 14 Q4	12	
6	Jun 16 Q3	12	
7	Jun 17 Q3	18	
8	Spec. Q3	14	
Total		**106**	

2. Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 5t - 14,$$

such that $x = 4\frac{1}{2}$ and $\frac{dx}{dt} = 3\frac{1}{2}$ when $t = 0$. [12]

3. Find the solution of the differential equation

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 18t - 87,$$

such that $x = 5$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 10$ when $t = 0$. [12]

3. Find the solution of the second order differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 6t + 5$$

such that $x = 3$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$ when $t = 0$. [12]

3. *(a)* A particle P, of mass 2 kg, moves along the horizontal x-axis under the action of a force directed towards the origin O. The magnitude of the force is equal to $8x$ N, where x m is the displacement of P from O. The particle is also subjected to a resistive force which is equal to $10v$ N, where v ms^{-1} is the speed of P at time t s. When $t = 0$ s, the particle P is at $x = 2$ m and it is moving away from O with speed 3 ms^{-1}.

(i) Show that the equation of motion of the particle is

$$\frac{d^2x}{dt^2} = -4x - 5\frac{dx}{dt}.$$

(ii) Find an expression for x in terms of t. [10]

(b) Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 12t - 3.$$ [4]

4. The reading x of the pointer on a set of kitchen scales at time t is modelled by the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 6\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 1.$$

(a) Find the general solution of the equation for x. [5]

(b) Determine the limiting value of x. [2]

(c) Given that x = 0·5 and $\frac{\mathrm{d}x}{\mathrm{d}t}$ = 0 when t = 0,

(i) find an expression for x in terms of t,

(ii) calculate the instantaneous reading of the scale when $t = \frac{\pi}{3}$.

Give your answer correct to three significant figures. [5]

3. Solve the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 6\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 27t,$$

where $x = \frac{\mathrm{d}x}{\mathrm{d}t} = 0$ when $t = 0$. Hence find the value of x when $t = 2$. [12]

3. The function x satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 6\frac{\mathrm{d}x}{\mathrm{d}t} + (10-k)x = \frac{1}{50}k(k-5)(12t-26),$$

where k is a constant. When $t = 0$, $x = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 16$. Find x in each of the following cases.

(a) $k = 5.$ [5]

(b) $k = 0.$ [5]

(c) $k = 10.$ [8]

3. A particle moves in a straight line such that at time t s, its displacement x m, from a fixed point O, satisfies the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 12x = 12t + 20 .$$

Given that when t = 0, x = 0 and the particle is moving with velocity 3 ms^{-1}, find its displacement at time t = 2 s. [14]