Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER MECHANICS A QUESTION PACK

0981-01 (Legacy M2) · New spec Unit 3 Topic 5 (2.3.3)

REVISE
.wales

FURTHER MATHS — MECH A · VERTICAL CIRCULAR MOTION

Vertical Circular Motion — Complete-Circle Conditions & Energy at the Top

Every vertical-circle question from the legacy WJEC M2 papers (June 2009–2017) — full loops on string / rod / inside-sphere, particle leaving the outside of a sphere.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours

*Derived from the legacy M2 paper's pace of **~1.5 min/mark** (80 marks over 7 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, narrowly focused on one sub-topic from Unit 3 (2.3.3). It contains every relevant question from the legacy WJEC M2 papers (2008 modular spec) that maps onto this sub-topic of new-spec AS Unit 3.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to. Take $g = 9.8$ m s^{-2} unless told otherwise.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 10 Q7	11	
2	Jun 11 Q8	13	
3	Jun 15 Q8	11	
4	Jun 17 Q6	12	

Q	Source	Max	Mark
5	Jun 14 Q7	12	
6	Jun 09 Q8	10	
7	Jun 16 Q9	11	
	Total	80	

Vertical Circular Motion — Complete-Circle Conditions & Energy at the Top — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 3: Further Mechanics A · Topic 2.3.3.

Vertical circle — energy 2.3.3

- Take the lowest point as the reference for h.
- By conservation of energy, speed at height h above lowest: $v^2 = u^2 - 2gh$.
- For radius r, $h = r(1 - \cos\theta)$ on the way up (string from above).
- At the top of the circle: $h = 2r$, so $v_{\text{top}}^2 = u^2 - 4gr$.

Tension at angle θ 2.3.3

- At angle θ below horizontal (string from above): $T - mg\cos\theta = \frac{mv^2}{r}$.
- Above the horizontal: $T + mg\cos\alpha = \frac{mv^2}{r}$ (with α from upward vertical).
- At the lowest point: $T = mg + \frac{mu^2}{r}$ (maximum tension).
- At the top of a full loop: $T = \frac{mv_{\text{top}}^2}{r} - mg$.

Conditions for complete circles 2.3.3

- **String:** need $T \geq 0$ at top. Critical $T = 0$ gives $v_{\text{top}}^2 = rg$, so $u_{\text{bot}}^2 \geq 5rg$.
- **Rod / rigid:** need $v_{\text{top}}^2 \geq 0$, so $u_{\text{bot}}^2 \geq 4rg$ – tension/thrust can adjust.
- **Inside smooth surface:** need $R \geq 0$ at top (same condition as string).
- If $u_{\text{bot}}^2 < 2gr$: particle oscillates as a pendulum (doesn't reach horizontal).

Leaving a smooth sphere 2.3.3

- Particle on outside of smooth sphere of radius r, starting at top.
- From energy: $v^2 = 2gr(1 - \cos\theta)$ at angle θ from upward vertical.
- Loses contact when normal reaction $R = 0$: $mg\cos\theta = \frac{mv^2}{r}$.
- Combine: $\cos\theta = \frac{2}{3}$ at the point of departure (from rest at top).

Vertical Circular Motion in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Energy in vertical circle

Take the lowest point as the reference for h.

By conservation of energy: $v^2 = u^2 - 2gh$ at height h.

For radius r: $h = r(1 - \cos\theta)$ on the lower half (θ from downward vertical).

At the top: $h = 2r$, so $v_{\text{top}}^2 = u^2 - 4gr$.

Tension at any angle

At angle θ from downward vertical (string from above), force toward centre:

$$T - mg\cos\theta = \frac{mv^2}{r}$$

Rearrange: $T = mg\cos\theta + \frac{mv^2}{r}$ on the lower semicircle.

Above the horizontal (α from upward vertical): $T + mg\cos\alpha = \frac{mv^2}{r}$.

Tension at lowest / highest

At the lowest point ($\theta = 0$):

$$T_{\text{bot}} = mg + \frac{mu^2}{r} \quad \text{(maximum)}$$

At the top of a full loop:

$$T_{\text{top}} = \frac{mv_{\text{top}}^2}{r} - mg \quad \text{(minimum)}$$

String — complete circle?

Need $T \geq 0$ at the top of the circle.

Critical case $T_{\text{top}} = 0$: $v_{\text{top}}^2 = rg$.

From energy: $u_{\text{bot}}^2 = v_{\text{top}}^2 + 4gr = 5rg$.

Condition: $u_{\text{bot}}^2 \geq 5rg$.

Rod — complete circle?

A rigid rod can *push* as well as pull — no $T \geq 0$ constraint.

Only need particle to reach the top with $v_{\text{top}}^2 \geq 0$.

Condition: $u_{\text{bot}}^2 \geq 4gr$.

Below this: oscillatory motion (pendulum-like).

Inside a smooth sphere

Particle on smooth concave inside of sphere of radius r.

Normal reaction R acts toward the centre, replacing tension.

Equation: $R - mg\cos\theta = \frac{mv^2}{r}$ (for θ below horizontal).

Same critical condition as string: $u_{\text{bot}}^2 \geq 5rg$ for complete loop.

Leaving the outside of a sphere

Particle on smooth outside of sphere, radius r, starting at top from rest.

From energy: $v^2 = 2gr(1 - \cos\theta)$.

Loses contact when $R = 0$: $mg\cos\theta = \frac{mv^2}{r}$.

Solve: $\cos\theta = \frac{2}{3}$ at the point of departure.

Common pitfalls

- Mixing up θ from downward vs upward vertical — check the sign of $\cos\theta$.
- Using $T \geq 0$ condition for a rod — only applies to strings / interior surfaces.
- Forgetting that v varies around the circle — not constant.
- Not checking the top of the circle when "complete revolution" is asked.

Strategy

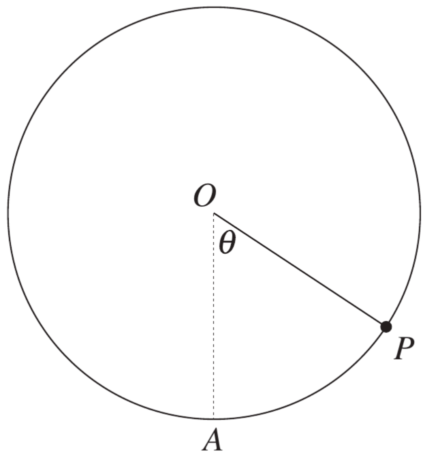
1. Mark the angle θ from the reference vertical on the diagram.
2. Energy: $v^2 = u^2 - 2gh(\theta)$.
3. Newton II radially: $T \pm mg\cos\theta = mv^2/r$.
4. For "complete circle": check $T \geq 0$ at top (string) or $v^2 \geq 0$ (rod).

SECTION T5

Vertical Circular Motion

Questions 1–7 · 80 marks

7. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 2·5 m to a fixed point O. Initially, P is projected from its lowest point A with a horizontal speed of $13\,\text{ms}^{-1}$ so that it starts to move in a vertical circle with centre O.



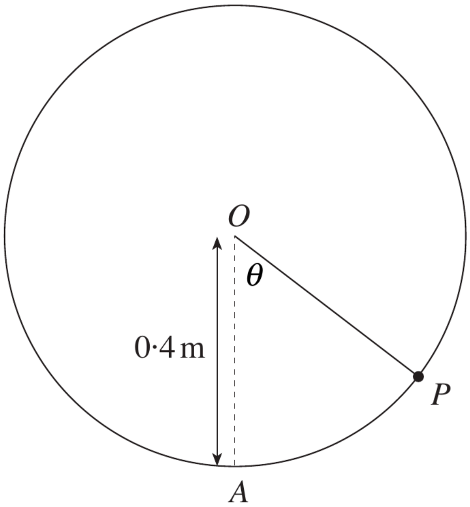
(a) Find an expression, in terms of θ, for the speed of P when OP makes an angle θ with OA.

Find the speed of P when $\cos\theta = \frac{1}{2}$. [5]

(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [2]

8. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 0·4 m to a fixed point O. Initially, P is projected from the point A, which is vertically below O, with a horizontal speed of 4 ms^{-1}.



(a) The speed of P when OP makes an angle θ with OA is v ms^{-1}.
Show that $v^2 = 8{\cdot}16 + 7{\cdot}84\cos\theta$. [4]

(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [3]

(d) Would your conclusion to *(c)* be different if the string was replaced by a light rigid rod? Justify your answer. [2]

8. One end of a light inextensible string of length 0·8 m is attached to a fixed point. The other end of the string is attached to a particle *P* of mass 3 kg. Initially *P* hangs at rest with the string vertical. The particle *P* is then projected horizontally with speed 5 $\mathrm{ms^{-1}}$, so that it starts to describe a vertical circle. When the string is inclined at an angle θ to the downwards vertical, *P* has speed v $\mathrm{ms^{-1}}$ and the tension in the string is T N.

(a) Find, in terms of θ,

(i) an expression for v^2,

(ii) an expression for T. [8]

(b) Find the greatest possible value of θ and briefly describe the subsequent motion of *P*. [3]

END OF PAPER

6. A particle P, of mass 5 kg, is attached to one end of a light inextensible string of length 0·8 m. The other end of the string is attached to a fixed point O. Initially, the particle P is held at rest with the string OP taut and inclined at an angle of 60° to the downward vertical through O. The particle P is then projected with speed $u\,\text{ms}^{-1}$ in a downward direction perpendicular to the string, so that P starts to describe a vertical circle with centre O. When the string OP is inclined at an angle θ to the downward vertical, the speed of P is $v\,\text{ms}^{-1}$.

(a) Find, in terms of u and θ, an expression for v^2. [4]

(b) Find, in terms of u and θ, an expression for the tension in the string when OP makes an angle θ with the downward vertical. [4]

(c) Determine the least value of u so that the particle describes complete circles. [2]

(d) Suppose that the string is replaced by a light rod. Determine the least value of u so that the particle describes complete circles. [2]

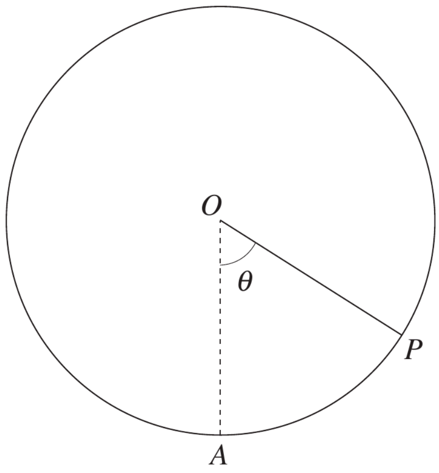
7. One end of a light rod of length l metres is freely jointed to a fixed point O and the other end is attached to a particle of mass m kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, u ms^{-1}, is a quarter of its speed at the lowest point of the circle.

(a) Show that $u^2 = \frac{4}{15}gl$. [3]

(b) When the rod is inclined at an angle θ to the **downward** vertical,
(i) find an expression for the tension in the rod in terms of m, g and θ.
(ii) determine the value of θ when the tension in the rod becomes zero. [9]

END OF PAPER

8. In the diagram below, A is the lowest point on the smooth inside surface of a sphere, with centre O and radius 2m. The point P is on the inside surface of the sphere such that $A\hat{O}P = \theta$. A particle, of mass 5 kg, is projected horizontally from A with speed 9 ms^{-1} so that it moves in the vertical circle with centre O which passes through P.



(a) Calculate, in terms of θ, the speed of the particle at P. [4]

(b) Find, in terms of θ, the reaction between the particle and the sphere at P. [4]

(c) Will the particle move in complete circles? Give a reason for your answer. [2]

9. A smooth sphere, with centre *O* and radius 4 m, is fixed. A particle *P*, of mass m, resting on the sphere at its highest point, is given a horizontal speed of magnitude $\sqrt{g}$ ms^{-1}, where g is the magnitude of the acceleration due to gravity. At the instant the line *OP* makes an angle θ with the upwards vertical, the speed of *P* is v ms^{-1}.

(a) Determine an expression for v^2 in terms of g and θ while *P* remains in contact with the sphere. [4]

(b) Find, in terms of m, g and θ, the magnitude of the force exerted by the sphere on *P*. Hence calculate the value of $\cos\theta$ and the value of v^2 when *P* leaves the surface of the sphere. [7]

END OF PAPER

END OF VERTICAL CIRCULAR MOTION PACK

Source: WJEC M2 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 3 — Topic 5 (2.3.3)