Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER MECHANICS A QUESTION PACK

0981-01 (Legacy M2) · New spec Unit 3 Topic 4

REVISE
.wales

FURTHER MATHS — MECH A · DIFFERENTIATION & INTEGRATION OF VECTORS

Vector Kinematics — Differentiation & Integration of Vectors

Every vector-kinematics question from the legacy WJEC M2 papers (June 2005 – June 2017 + Specimen). The vector-calculus strand is treated heavily by M2 rather than M3 in the legacy 2008 specification.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~4 hours 58 minutes

*Derived from the legacy M2 paper's pace of **~1.5 min/mark** (199 marks over 21 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every differentiation & integration of vectors question from the legacy WJEC M2 papers (2008 modular spec) that maps onto new-spec AS Unit 3 Topic 4 (2.3.4).

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to. Take $g = 9.8$ m s^{-2} unless told otherwise.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 07 Q5	6	
2	Jun 08 Q6	5	
3	Jun 15 Q1	6	
4	Jun 06 Q6	9	
5	Jun 11 Q3	7	
6	Jun 12 Q3	8	
7	Jun 10 Q2	13	
8	Jun 13 Q2	12	
9	Jun 09 Q6	11	
10	Jun 16 Q6	7	
11	Jun 14 Q6	13	
12	Jun 17 Q1	11	
13	Jun 06 Q2	5	
14	Jun 07 Q8	10	
15	Jun 11 Q7	10	
16	Jun 14 Q4	10	
17	Jun 16 Q3	9	
18	Jun 08 Q7	10	
19	Jun 12 Q8	10	
20	Jun 05 Q6	11	
21	Spec. Q5	16	
	Total	199	

Vector Kinematics — Differentiation & Integration of Vectors — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 3: Further Mechanics A · Topic 2.3.4.

Vector kinematics 2.3.4

- Position vector in 2D/3D: $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$.
- Velocity: $\mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt}$.
- Acceleration: $\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}}$.
- Speed $= |\mathbf{v}|$; momentum $\mathbf{p} = m\mathbf{v}$; force $\mathbf{F} = m\mathbf{a}$.

Differentiation & integration 2.3.4

- Differentiate component-wise: $\dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$.
- Integrate with initial conditions to recover $\mathbf{v}(t)$ from $\mathbf{a}(t)$, $\mathbf{r}(t)$ from $\mathbf{v}(t)$.
- Perpendicularity at time t: $\mathbf{u} \cdot \mathbf{v} = 0$. Parallel: $\mathbf{u} = \lambda\mathbf{v}$.
- Power instantaneously delivered by force $\mathbf{F}$ on a particle of velocity $\mathbf{v}$: $P = \mathbf{F} \cdot \mathbf{v}$.

Relative motion & shortest distance 2.3.4

- Relative position: $\mathbf{r}_{AB} = \mathbf{r}_A - \mathbf{r}_B$.
- Relative velocity: $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$.
- Distance between A and B at time t: $|\mathbf{r}_{AB}(t)|$ — minimise $|\mathbf{r}_{AB}|^2$ over t.
- Set $\frac{d}{dt}|\mathbf{r}_{AB}|^2 = 0$ and solve for t^*; substitute back for the least distance.

Working scientifically general

- Always integrate *component-wise* and include a constant of integration for each component.
- Use initial conditions ($\mathbf{r}(0)$, $\mathbf{v}(0)$) to fix the constants — don't skip this step.
- For shortest distance: minimise $|\mathbf{r}_{AB}|^2$ (avoids square roots) — algebraically tidier.
- For perpendicularity: use $\mathbf{u} \cdot \mathbf{v} = 0$. Show the dot product expands and solve for t.

Differentiation & Integration of Vectors in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Position, velocity, accel

Position vector $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$.

$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$.

$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}}$.

Differentiate / integrate *each component* separately.

Speed, momentum, force

Speed (a scalar): $|\mathbf{v}| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$.

Momentum vector: $\mathbf{p} = m\mathbf{v}$.

Newton II in vector form: $\mathbf{F} = m\mathbf{a}$.

Magnitude of force: $|\mathbf{F}| = m|\mathbf{a}|$.

Integration with initial conds

Given $\mathbf{a}(t)$, integrate component-wise:

$\mathbf{v}(t) = \mathbf{v}(0) + \int_0^t \mathbf{a}\,du$.

$\mathbf{r}(t) = \mathbf{r}(0) + \int_0^t \mathbf{v}\,du$.

Each scalar component gets its own constant of integration — pin from $\mathbf{v}(0), \mathbf{r}(0)$.

Perpendicular & parallel

Vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular iff $\mathbf{u} \cdot \mathbf{v} = 0$.

They are parallel iff $\mathbf{u} = \lambda\mathbf{v}$ for some scalar λ.

In 2D: $(a\mathbf{i} + b\mathbf{j}) \cdot (c\mathbf{i} + d\mathbf{j}) = ac + bd$.

Work & power

Work done by constant force $\mathbf{F}$ along displacement $\mathbf{d}$:

$$W = \mathbf{F} \cdot \mathbf{d}$$

Instantaneous power:

$$P = \mathbf{F} \cdot \mathbf{v}$$

Relative position & velocity

$\mathbf{r}_{AB} = \mathbf{r}_A - \mathbf{r}_B$ (position of A relative to B).

$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$.

If both are at constant velocity: $\mathbf{r}_{AB}(t) = \mathbf{r}_{AB}(0) + t\,\mathbf{v}_{AB}$.

Shortest distance

Distance: $|\mathbf{r}_{AB}(t)|$. Minimise its square: $f(t) = |\mathbf{r}_{AB}(t)|^2 = \mathbf{r}_{AB} \cdot \mathbf{r}_{AB}$.

Set $f'(t) = 0$ — gives $\mathbf{r}_{AB} \cdot \mathbf{v}_{AB} = 0$ — substitute back for distance.

Common pitfalls

- Forgetting initial conditions when integrating — each component needs its own C.
- Computing speed instead of velocity (vector) or vice versa.
- Mixing 2D and 3D in dot products — missing components default to 0.
- Minimising $|\mathbf{r}_{AB}|$ instead of its square — square is algebraically cleaner.

Strategy

1. Write $\mathbf{r}_A(t), \mathbf{r}_B(t)$ explicitly.
2. Differentiate / integrate component-wise.
3. For perpendicularity: dot product $=$ 0, solve for t.
4. For closest approach: minimise $|\mathbf{r}_{AB}|^2$ over t.

SECTION T4

Differentiation & Integration of Vectors

Questions 1–21 · 199 marks

5. Vectors **a** and **b** are given by

$$\mathbf{a} = 2\mathbf{i} + 13\mathbf{j} - 10\mathbf{k},$$
$$\mathbf{b} = -\mathbf{i} + y\mathbf{j} + 5\mathbf{k}.$$

(a) If **a** and **b** are perpendicular, find the value of y. [4]

(b) If **a** and **b** are parallel, find the value of y. [2]

6. A constant force $\mathbf{F} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$ acts on a bead as it moves along a straight smooth wire from point A to point B. Point A has position vector $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and point B has position vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find

(a) the vector $\mathbf{AB}$, [2]

(b) the work done by the force $\mathbf{F}$. [3]

1. The vectors $\mathbf{x}$ and $\mathbf{y}$ are given by

$$\mathbf{x} = \sin\theta\,\mathbf{i} + 2\cos 2\theta\,\mathbf{j},$$
$$\mathbf{y} = 2\mathbf{i} - \mathbf{j}.$$

Find the values of θ between 0 and 2π such that $\mathbf{x}$ is perpendicular to $\mathbf{y}$. [6]

6. A particle P moves such that its position vector $\mathbf{r}$ with respect to the origin O at time t is given by

$$\mathbf{r} = \cos 3t\mathbf{i} + \sin 3t\mathbf{j}.$$

(a) Find an expression for $\mathbf{v}$, the velocity of P at time t. [3]

(b) Show that the direction of $\mathbf{v}$ is perpendicular to that of $\mathbf{r}$ for all values of t. [3]

(c) Find the speed of P. [3]

3. A particle P, of mass 2 kg, is moving under the action of a force **F** N so that its velocity **v** ms^{-1} at time t s is given by

$$\mathbf{v} = 2\mathbf{i} + 6t\mathbf{j} + 4t^3\mathbf{k}.$$

(a) Find an expression for **F** at time t s. [3]

(b) Determine the value of **F.v** when $t = 1$ and state the units of your answer. [4]

3. A particle moves on a horizontal plane so that at time t seconds its position vector $\mathbf{r}$ metres relative to a fixed origin O is given by

$$\mathbf{r} = (t + 2t^2)\mathbf{i} + (1{\cdot}5t^2 - 2t)\mathbf{j}.$$

(a) Determine the time when the velocity of the particle is perpendicular to the vector $(-\mathbf{i} + 2\mathbf{j})$. [5]

(b) Show that the acceleration of the particle is constant and find its magnitude. [3]

2. At time t s, the position vector $\mathbf{r}$ m of a particle P is given by

$$\mathbf{r} = (3t^2 + 1)\mathbf{i} + (13t - 2t^2)\mathbf{j}.$$

(a) Find the speed of P when $t = 2$. [4]

(b) Calculate the value of t when the velocity of P is perpendicular to the vector $2\mathbf{i} - \mathbf{j}$. [3]

(c) Show that the acceleration of P is constant and find its magnitude. [3]

(d) Find the angle between the direction of the acceleration of P and the direction of the velocity of P when $t = 2$. [3]

2. A particle P, of mass 2 kg, is moving so that at time t s its velocity $\mathbf{v}$ ms^{-1} is given by $\mathbf{v} = (13t - 3)\mathbf{i} + (2 + 3t^2)\mathbf{j}$. At time $t = 0$ s, the position vector of the particle is $(2\mathbf{i} + 7\mathbf{j})$ m.

(a) Find the position vector $\mathbf{r}$ of P at time t s. [5]

(b) Determine the acceleration $\mathbf{a}$ of P at time t s. [2]

(c) Calculate the values of t when the velocity of P is perpendicular to the vector $\mathbf{i} - 2\mathbf{j}$. [5]

6. A particle, of mass 2 kg, moves in a horizontal plane such that its position vector **r** m at time t s is given by

$$\mathbf{r} = (1 - 4t^2)\,\mathbf{i} + (3t^2 - 5t)\mathbf{j}.$$

(a) Find, in terms of t, an expression for the momentum of the particle at time t s. [3]

(b) Show that the acceleration of the particle is constant and find its magnitude. [4]

(c) Find the time when the velocity of the particle is perpendicular to its acceleration. [4]

6. A particle moves on a horizontal plane such that its velocity vector $\mathbf{v}$ ms^{-1} at time t s is given by

$$\mathbf{v} = 7\sin 2t\,\mathbf{i} + 6\cos 3t\,\mathbf{j}.$$

(a) Find the acceleration vector of the particle at time t s. [2]

(b) Given that when $t = 0$, the particle has position vector $(0{\cdot}5\mathbf{i} + 3\mathbf{j})$ m, find the position vector of the particle when $t = \dfrac{\pi}{2}$. [5]

6. A particle of mass 3 kg moves on a horizontal plane. At time $t = 0$, the particle has position vector $-2\mathbf{i} + 3\mathbf{j}$ m, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the x-axis and y-axis respectively. At time t s, the particle moves with velocity $\mathbf{v}$ ms^{-1} given by

$$\mathbf{v} = 4\sin 2t\,\mathbf{i} + 15\cos 5t\,\mathbf{j}.$$

(a) Find the magnitude of the force acting on the particle at time $t = \frac{3\pi}{2}$ s. [5]

(b) Determine the position vector of the particle at time t s. [4]

(c) Calculate the time and the distance of the particle from the origin when it crosses the y-axis for the first time. [4]

1. The position vector of a particle P at time t seconds is given by

$$\mathbf{r} = t\sin t\,\mathbf{i} + t\cos t\,\mathbf{j}.$$

(a) (i) Find the velocity vector of P and an expression for the speed of P at time t seconds in its simplest form.

(ii) Given that the mass of P is 3 kg, write down the momentum vector of P at time t seconds. [6]

(b) At time $t = \frac{\pi}{6}$, the vector $b\mathbf{i} + \sqrt{3}\mathbf{j}$ is perpendicular to $\mathbf{r}$. Find the value of b. [5]

2. Particle A is moving with constant velocity $-2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$, and at time $t = 0$ s it has position vector $\mathbf{i} - 10\mathbf{k}$. Particle B is moving with constant velocity $\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}$, and at time $t = 0$ s it has position vector $7\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$.

(a) Write down the position vectors of A and B at time t s. [2]

(b) Find the distance between A and B when $t = 2$ s. [3]

8. A toy plane A is moving with constant velocity $(3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})\,\text{ms}^{-1}$ and at time $t = 0$, its position vector is $(3\mathbf{j} - 140\mathbf{k})$ m. Another toy plane B is moving with constant velocity $(-2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})\,\text{ms}^{-1}$ and at time $t = 0$, its position vector is $(-9\mathbf{i} - 4\mathbf{j} - 6\mathbf{k})$ m.

(a) Write down the position vectors of A and B at time t s. [3]

(b) Find an expression for the square of the distance between A and B at time t s. [3]

(c) Determine the time when A and B are closest together. [4]

7. At time t, the position vectors relative to a fixed origin O, of two particles A and B are given by $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k})$ and $\mathbf{OB} = 5\mathbf{i} - 8\mathbf{j} + 10\mathbf{k} + t(3\mathbf{i} - 6\mathbf{j} + 7\mathbf{k})$.

(a) Find the speed of particle A. [3]

(b) Show that the distance AB at time t is given by $AB^2 = 5t^2 - 30t + 211$. Determine the time at which the particles A and B are closest together. [7]

4. At time t = 0, an aeroplane A has position vector $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})$ m and is flying with constant velocity $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ ms^{-1}.
At time t = 0, another aeroplane B has position vector $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})$ m, and is flying with constant velocity $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ ms^{-1}.

(a) Find expressions for the position vector of A and the position vector of B at time t s. [3]

(b) Determine an expression for AB^2, where AB is the distance between A and B at time t s. [4]

(c) Given that the shortest distance between A and B occurs at t = 5, calculate the value of x. [3]

3. At time $t = 0$ s, the position vector of an object *A* is $\mathbf{i}$ m and the position vector of another object *B* is $3\mathbf{i}$ m. The constant velocity vector of *A* is $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ ms^{-1} and the constant velocity vector of *B* is $\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ ms^{-1}. Determine the value of t when *A* and *B* are closest together and find the least distance between *A* and *B*. [9]

7. *(a)* A vehicle moves with velocity $\mathbf{v} = \sin(3t)\mathbf{i} + 2\cos(5t)\mathbf{j} + 3t^3\mathbf{k}$ at time t. Find an expression for the acceleration of the vehicle at time t. [3]

(b) Two vehicles A and B move on the same horizontal plane. At time t, A is at position $(-8t - 2)\mathbf{i} + (3t + 3)\mathbf{j}$ and B is at position $(-16t + 11)\mathbf{i} + (9t - 8)\mathbf{j}$. Determine the value of t when the distance between A and B is least, and calculate this distance. [7]

8. A ship S is moving in a straight line with constant velocity. At time $t = 0$, its position vector relative to a fixed origin O is $(8\mathbf{i} + 7\mathbf{j})$. At time $t = 3$, its position vector is $(14\mathbf{i} - 5\mathbf{j})$.

(a) Show that the velocity of S is $(2\mathbf{i} - 4\mathbf{j})$. [2]

(b) Find an expression, in terms of t, for the position vector of S at time t. [2]

At time $t = 10$, a boat B leaves O and travels with constant velocity $x\mathbf{i} + y\mathbf{j}$, intercepting S at time $t = 50$.

(c) Calculate the value of x and the value of y. [6]

6. At time t s, a particle P has position vector $\mathbf{r}$ m with respect to an origin O given by

$$\mathbf{r} = (2t - 5)\mathbf{i} + (t - 3)\mathbf{j} + (7 - 2t)\mathbf{k}.$$

(a) Show that the distance of the particle from the origin at time t s is given by

$$OP^2 = 9t^2 - 54t + 83,$$

and find the value of t when P is closest to O. [5]

(b) Find the velocity of P and determine its magnitude. [3]

(c) Show that, when P is closest to O, the direction of the velocity of P is perpendicular to the line OP. [3]

5. A particle moves with constant acceleration. Initially, the particle is moving with velocity (i + 2j) ms^{-1}. The velocity of the particle at t = 2 s is (3i – 2j) ms^{-1}.

(a) Show that its acceleration is i – 2j. [2]

(b) Find the velocity of the particle at time t s. [4]

(c) Determine the time when the velocity vector is perpendicular to the acceleration vector. [3]

(d) Given that the initial position vector of the particle is (2i – j)m, find the position vector of the particle at time t. [4]

(e) Evaluate the distance of the particle from the origin at time t = 2 s. [3]

END OF DIFFERENTIATION & INTEGRATION OF VECTORS PACK

Source: WJEC M2 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 3 — Topic 4 (2.3.4)