Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER MECHANICS A QUESTION PACK

0981-01 (Legacy M2) · New spec Unit 3 Topic 3

REVISE
.wales

FURTHER MATHS — MECH A · CIRCULAR MOTION

Circular Motion — Horizontal & Vertical Circles

Every circular-motion question from the legacy WJEC M2 papers (June 2005 – June 2017 + Specimen) covering horizontal circles, conical pendulums, banked tracks and full vertical-circle revolutions.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~4 hours 2 minutes

*Derived from the legacy M2 paper's pace of **~1.5 min/mark** (161 marks over 18 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every circular motion question from the legacy WJEC M2 papers (2008 modular spec) that maps onto new-spec AS Unit 3 Topic 3 (2.3.3).

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to. Take $g = 9.8$ m s^{-2} unless told otherwise.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q2	4	
2	Jun 07 Q6	4	
3	Jun 15 Q7	5	
4	Jun 09 Q7	7	
5	Jun 10 Q6	7	
6	Jun 16 Q8	7	
7	Jun 06 Q7	7	
8	Jun 12 Q7	8	
9	Spec. Q7	9	

Q	Source	Max	Mark
10	Jun 13 Q4	9	
11	Jun 17 Q7	14	
12	Jun 10 Q7	11	
13	Jun 11 Q8	13	
14	Jun 15 Q8	11	
15	Jun 17 Q6	12	
16	Jun 14 Q7	12	
17	Jun 09 Q8	10	
18	Jun 16 Q9	11	
Total		161	

Circular Motion — Horizontal & Vertical Circles — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 3: Further Mechanics A · Topic 2.3.3.

Horizontal circular motion 2.3.3

- For motion in a circle of radius r with speed v and angular speed ω: $v = r\omega$.
- Centripetal acceleration toward the centre: $a = \frac{v^2}{r} = r\omega^2$.
- Net horizontal force toward centre: $F = \frac{mv^2}{r} = mr\omega^2$.
- In horizontal-circle problems vertical equilibrium gives one equation; centripetal gives the other.

Conical pendulum & banked tracks 2.3.3

- Conical pendulum (mass on string length L at angle θ to vertical): $\tan\theta = \frac{v^2}{rg} = \frac{r\omega^2}{g}$.
- Banked track (no friction): $\tan\theta = \frac{v^2}{rg}$ gives the "design speed".
- With friction μ and design speed exceeded: $\tan\theta + \mu \geq \frac{v^2}{rg}(1 - \mu\tan\theta)$.
- Particle on rough horizontal disc: $F_{\text{friction}} = \mu R = \frac{mv^2}{r}$ at the slipping point.

Vertical circular motion 2.3.3

- Use conservation of energy to relate speed at height h above lowest point: $v^2 = u^2 - 2gh$.
- Tension / reaction at angle θ from vertical: $T - mg\cos\theta = \frac{mv^2}{r}$.
- On a string / inside a hollow sphere: complete circles need $v_{\text{top}} \geq \sqrt{rg}$ (so $T \geq 0$).
- On a rod (rigid) or inside smooth hemisphere: $v_{\text{top}} \geq 0$ — tension/thrust can adjust.
- On outside of a sphere: particle leaves when normal reaction $R = 0$ — gives angle of departure.

Working scientifically general

- Sketch a force diagram *before* writing equations — identify which forces have centripetal components.
- Resolve vertically *and* radially (toward centre) — two independent equations.
- For "complete revolution" questions: check the critical condition at the top of the circle.
- Energy and Newton's second law are usually *both* needed — energy for speed, $F = mv^2/r$ for tension.

Circular Motion in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Circular kinematics

For circular motion of radius r with speed v and angular speed ω:

$$v = r\omega$$

Centripetal acceleration toward the centre:

$$a = \frac{v^2}{r} = r\omega^2$$

Centripetal force

Net force toward centre:

$$F = \frac{mv^2}{r} = mr\omega^2$$

This is not a "new" force — it's the resultant of tension / friction / gravity components pointing inward.

Conical pendulum

Mass on light string length L at angle θ to vertical, moving in horizontal circle radius $r = L\sin\theta$.

Vertical: $T\cos\theta = mg$.

Horizontal: $T\sin\theta = \frac{mv^2}{r}$.

Divide: $\tan\theta = \frac{v^2}{rg}$.

Banked track (no friction)

Track banked at angle θ, vehicle in horizontal circle radius r.

For "design speed" (no friction needed):

$$\tan\theta = \frac{v^2}{rg}$$

Normal reaction provides both vertical and horizontal components.

Vertical circle — energy

Take the lowest point as the reference for h.

By conservation of energy, speed at height h above lowest:

$$v^2 = u^2 - 2gh$$

For radius r, $h = r(1 - \cos\theta)$ on the way up.

Vertical circle — tension

At angle θ from downward vertical (string from above):

$$T - mg\cos\theta = \frac{mv^2}{r}$$

Rearrange: $T = mg\cos\theta + \frac{mv^2}{r}$ on the lower semicircle.

Above the horizontal: $T + mg\cos\alpha = \frac{mv^2}{r}$ (with α from upward vertical).

Conditions for complete circles

String: need $T \geq 0$ at top. Critical: $T = 0$ gives $v_{\text{top}}^2 = rg$, i.e. $u_{\text{bot}}^2 \geq 5rg$.

Rod / rigid: need $v_{\text{top}}^2 \geq 0$, i.e. $u_{\text{bot}}^2 \geq 4rg$.

Inside smooth surface: same as string (need $R \geq 0$).

Particle leaving a sphere

Particle slides on outside of smooth sphere of radius r from the top:

$v^2 = 2gr(1 - \cos\theta)$ from energy.

Loses contact when $R = 0$: $mg\cos\theta = \frac{mv^2}{r}$.

Solve: $\cos\theta = \frac{2}{3}$ at the point of departure (from rest).

Strategy

1. Draw force diagram; identify which forces have centripetal components.
2. Resolve vertically (equilibrium) and radially ($= mv^2/r$).
3. For vertical circles, use energy conservation for v, then $F = mv^2/r$ for T.
4. For "complete revolution": check critical condition at the top.

SECTION T3

Circular Motion

Questions 1–18 · 161 marks

2. A particle of mass 0·5 kg is attached to one end of a light inextensible string of length 0·6 m. The other end of the string is fixed at a point O on a smooth horizontal surface. The particle moves on the surface in a circle with centre O, so that the string is taut and the angular velocity of the particle about O is 5 radians per second.

(a) Calculate the speed of the particle. [2]

(b) Find the tension in the string. [2]

6. A particle of mass 0·8 kg is attached to one end of a light inextensible string of length 0·4 m. The other end of the string is fixed to a point O of a smooth horizontal surface. The particle moves on the surface with constant speed $3\,\text{ms}^{-1}$ in a horizontal circle with centre O.

(a) Find the angular velocity about O of the particle. [2]

(b) Calculate the tension in the string. [2]

TURN OVER

7. A car of mass 1200 kg is moving in a horizontal circle of radius 80 m on a road banked at an angle of 12° to the horizontal.
When the car is moving with a constant speed of v ms^{-1}, there is no tendency to sideslip. Calculate the normal reaction of the road on the car and find the value of v. [5]

TURN OVER

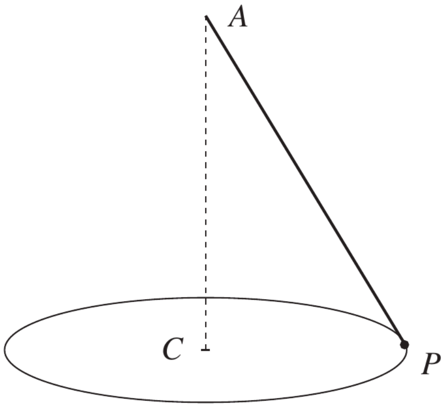
7. A car, of mass 1000 kg, is travelling in a horizontal circle of radius 250 m on a track which is banked at an angle α to the horizontal. When the car is travelling at 28 ms^{-1}, it has no tendency to slip sideways. Calculate the value of α. [7]

TURN OVER

6. An athlete is cycling at a constant speed $v\,\text{ms}^{-1}$ in a horizontal circle, of radius 40 m, on a track that is banked at an angle of 30° to the horizontal. The combined mass of the bicycle and the athlete is 60 kg and the coefficient of friction between the bicycle tyres and the track is $\frac{1}{4}$. Find, correct to three significant figures, the greatest possible value of v. [7]

8. A rough circular plate rotates horizontally about a smooth fixed vertical axis through its centre *O*. A point *A* on the plate moves with constant speed $v\,\text{ms}^{-1}$, where *OA* is 1·6 m. A particle of mass m kg lies on the point *A* on the plate. The coefficient of friction between the particle and the plate is 0·72. Given that the particle remains at the point *A*, find the greatest possible value of v. Hence write down the greatest possible value of the angular velocity of the particle. State clearly your units for the angular velocity. [7]

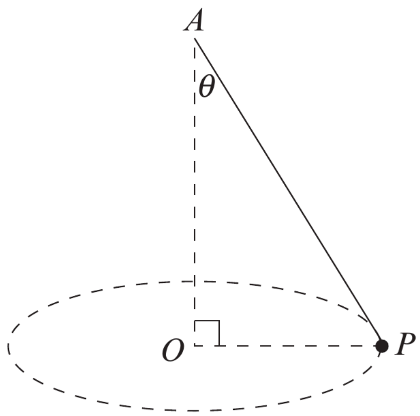
7. The diagram shows a small body P, of mass 3 kg, attached by means of a light inextensible string, of length 1·3 m, to a fixed point A. The point C is vertically below A, and P describes a horizontal circle, with centre C and radius 0·5 m, with a uniform angular speed of ω radians per second about C.



(a) Find the tension in the string. [3]

(b) Calculate, correct to two decimal places, the value of ω. [4]

7. One end of a light inextensible string is attached to a fixed point A. The other end is attached to a particle P of mass 3 kg. The point O is vertically below A and P moves in a horizontal circle of centre O with a uniform angular speed of 2·8 radians per second. The tension in the string is 88·2 N and $O\hat{A}P$ is θ.



(a) Find the value of θ. [3]

(b) Calculate the length of the string. [5]

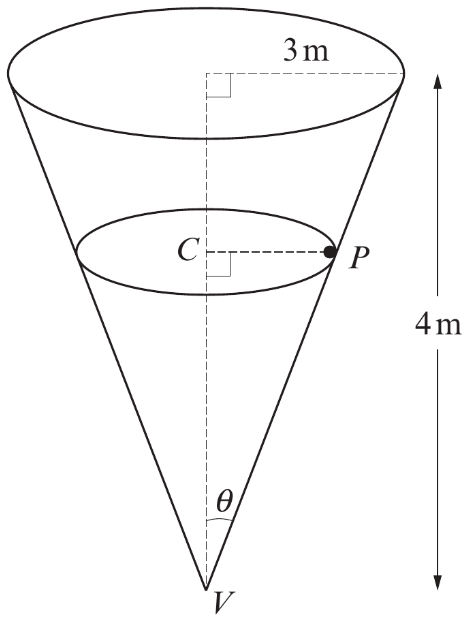
fixed vertical pole. The other end B is attached to a small ball of mass 0·2 kg. A boy holds the ball so that the rope makes an angle of 30° with the pole. He then hits the ball so that the point B moves with speed u ms^{-1} in a horizontal circle, with the rope remaining at 30° to the vertical throughout the motion.

(a) Calculate the magnitude of the tension in the rope. [3]

(b) Find the value of u, correct to two decimal places. [5]

(c) What assumption does the word 'light' enable you to make in your solution. [1]

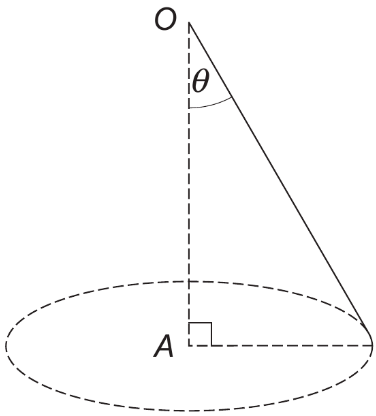
4. The diagram shows a hollow cone, of base radius 3 m and height 4 m, which is fixed with its axis vertical and vertex V downwards. A particle P, of mass M kg, moves in the horizontal circle with centre C on the smooth inner surface of the cone with constant speed $\sqrt{\frac{8g}{3}}$ ms^{-1}, where g ms^{-2} is the acceleration due to gravity.



(a) Show that the normal reaction of the surface of the cone on the particle is $\frac{5Mg}{3}$ N. [4]

(b) Calculate the length of CP and hence determine the height of C above V. [5]

7. A particle of mass 2 kg is suspended from a fixed point *O* by means of an elastic string of natural length 3 m and modulus of elasticity λ N. The particle describes a horizontal circle with constant angular speed ω rad s^{-1}, with the string being of constant length l m, where $l > 3$. The centre of the circle *A* is vertically below *O* and the angle between the string and the downward vertical is θ.



(a) Show that $\cos\theta = \dfrac{g}{l\omega^2}$. [6]

(b) Given that the tension in the string is $20g$ N and $\omega^2 = 3g$,

(i) find the value of $\cos\theta$,

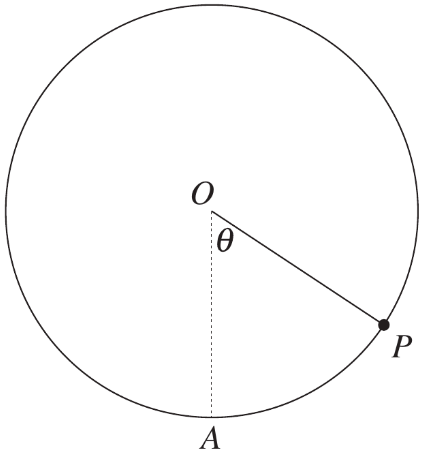
(ii) show that $l = \dfrac{10}{3}$,

(iii) calculate the value of λ,

(iv) find the elastic energy in the string. [8]

END OF PAPER

7. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 2·5 m to a fixed point O. Initially, P is projected from its lowest point A with a horizontal speed of $13\,\text{ms}^{-1}$ so that it starts to move in a vertical circle with centre O.



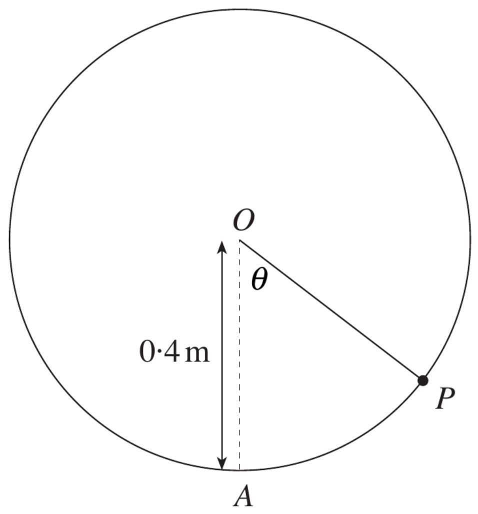
(a) Find an expression, in terms of θ, for the speed of P when OP makes an angle θ with OA.

Find the speed of P when $\cos\theta = \frac{1}{2}$. [5]

(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [2]

8. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 0·4 m to a fixed point O. Initially, P is projected from the point A, which is vertically below O, with a horizontal speed of $4\,\text{ms}^{-1}$.



(a) The speed of P when OP makes an angle θ with OA is $v\,\text{ms}^{-1}$.
Show that $v^2 = 8{\cdot}16 + 7{\cdot}84\cos\theta$. [4]

(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [3]

(d) Would your conclusion to *(c)* be different if the string was replaced by a light rigid rod? Justify your answer. [2]

8. One end of a light inextensible string of length 0·8 m is attached to a fixed point. The other end of the string is attached to a particle *P* of mass 3 kg. Initially *P* hangs at rest with the string vertical. The particle *P* is then projected horizontally with speed 5 ms^{-1}, so that it starts to describe a vertical circle. When the string is inclined at an angle θ to the downwards vertical, *P* has speed v ms^{-1} and the tension in the string is T N.

(a) Find, in terms of θ,

(i) an expression for v^2,

(ii) an expression for T. [8]

(b) Find the greatest possible value of θ and briefly describe the subsequent motion of *P*. [3]

END OF PAPER

6. A particle P, of mass 5 kg, is attached to one end of a light inextensible string of length 0·8 m. The other end of the string is attached to a fixed point O. Initially, the particle P is held at rest with the string OP taut and inclined at an angle of 60° to the downward vertical through O. The particle P is then projected with speed u ms^{-1} in a downward direction perpendicular to the string, so that P starts to describe a vertical circle with centre O. When the string OP is inclined at an angle θ to the downward vertical, the speed of P is v ms^{-1}.

(a) Find, in terms of u and θ, an expression for v^2. [4]

(b) Find, in terms of u and θ, an expression for the tension in the string when OP makes an angle θ with the downward vertical. [4]

(c) Determine the least value of u so that the particle describes complete circles. [2]

(d) Suppose that the string is replaced by a light rod. Determine the least value of u so that the particle describes complete circles. [2]

7. One end of a light rod of length l metres is freely jointed to a fixed point O and the other end is attached to a particle of mass m kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, u ms^{-1}, is a quarter of its speed at the lowest point of the circle.

(a) Show that $u^2 = \frac{4}{15}gl$. [3]

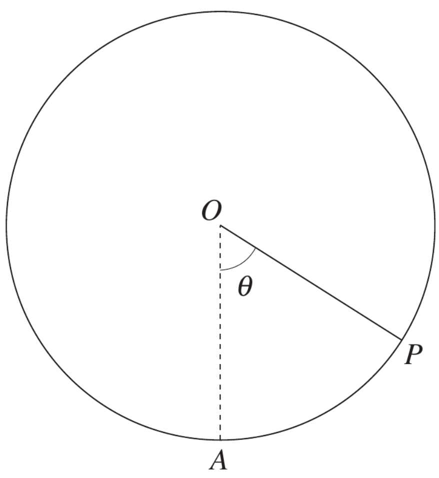
(b) When the rod is inclined at an angle θ to the **downward** vertical,

(i) find an expression for the tension in the rod in terms of m, g and θ.

(ii) determine the value of θ when the tension in the rod becomes zero. [9]

END OF PAPER

8. In the diagram below, A is the lowest point on the smooth inside surface of a sphere, with centre O and radius 2m. The point P is on the inside surface of the sphere such that $A\hat{O}P = \theta$. A particle, of mass 5 kg, is projected horizontally from A with speed 9 ms^{-1} so that it moves in the vertical circle with centre O which passes through P.



(a) Calculate, in terms of θ, the speed of the particle at P. [4]

(b) Find, in terms of θ, the reaction between the particle and the sphere at P. [4]

(c) Will the particle move in complete circles? Give a reason for your answer. [2]

9. A smooth sphere, with centre *O* and radius 4 m, is fixed. A particle *P*, of mass m, resting on the sphere at its highest point, is given a horizontal speed of magnitude $\sqrt{g}$ ms^{-1}, where g is the magnitude of the acceleration due to gravity. At the instant the line *OP* makes an angle θ with the upwards vertical, the speed of *P* is v ms^{-1}.

(a) Determine an expression for v^2 in terms of g and θ while *P* remains in contact with the sphere. [4]

(b) Find, in terms of m, g and θ, the magnitude of the force exerted by the sphere on *P*. Hence calculate the value of $\cos\theta$ and the value of v^2 when *P* leaves the surface of the sphere. [7]

END OF PAPER

END OF CIRCULAR MOTION PACK

Source: WJEC M2 (2008 modular spec) · 2005–2017

Curated for WJEC FM 2017 spec AS Unit 3 — Topic 3 (2.3.3)