Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER MECHANICS A QUESTION PACK

0981-01 (Legacy M2) · New spec Unit 3 Topic 2

REVISE
.wales

FURTHER MATHS — MECH A · HOOKE'S LAW, WORK, ENERGY & POWER

Hooke's Law, Work, Energy & Power

Every Hooke / energy / power question from the legacy WJEC M2 papers (June 2005 – June 2017 + Specimen) that maps onto new-spec AS Unit 3 Topic 2.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 44 minutes

*Derived from the legacy M2 paper's pace of **~1.5 min/mark** (109 marks over 13 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every hooke's law, work, energy & power question from the legacy WJEC M2 papers (2008 modular spec) that maps onto new-spec AS Unit 3 Topic 2 (2.3.2).

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to. Take $g = 9.8$ m s^{-2} unless told otherwise.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 05 Q1	5	
2	Jun 08 Q1	6	
3	Jun 12 Q2	5	
4	Jun 06 Q4	7	
5	Jun 07 Q3	12	
6	Jun 10 Q3	11	
7	Jun 11 Q5	10	

Q	Source	Max	Mark
8	Jun 13 Q7	11	
9	Jun 15 Q5	11	
10	Jun 16 Q5	6	
11	Jun 09 Q2	8	
12	Jun 14 Q1	7	
13	Spec. Q3	10	
Total		109	

Hooke's Law, Work, Energy & Power — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 3: Further Mechanics A · Topic 2.3.2.

Hooke's law 2.3.2

- For a light elastic string / spring of natural length l and modulus of elasticity λ:
- Tension when extended by x: $T = \frac{\lambda x}{l}$.
- A spring resists compression as well as extension; a string only resists extension (slack otherwise).
- In equilibrium: $T = mg$ gives $\frac{\lambda x_0}{l} = mg$ for the natural extension.

Elastic potential energy 2.3.2

- EPE stored in a string / spring at extension x: $\text{EPE} = \frac{\lambda x^2}{2l}$.
- EPE is positive whether stretched or (for a spring) compressed.
- No EPE when the string is at its natural length or slack.
- Derivable from $W = \int T\,dx$.

Work, energy & power 2.3.2

- Work-energy theorem: total work done $W = \Delta KE$.
- Conservation of energy: $\sum E_i = \sum E_f$ when only conservative forces act.
- KE $= \frac{1}{2}mv^2$; GPE $= mgh$ (relative to a reference level).
- Power: $P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$ — for a force in the direction of motion, $P = Fv$.

Working scientifically general

- For "released from rest, find speed" problems: equate *initial* total energy to *final* total energy.
- Don't double-count EPE — only include it when the string is stretched.
- For motion under gravity + spring: include KE, GPE and EPE in the energy equation.
- For a vehicle at constant speed, driving force $\times$ speed $=$ engine power; balance with resistance.

Hooke's Law, Work, Energy & Power in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Hooke's law

For a light elastic string / spring with modulus λ and natural length l, extension x:

$$T = \frac{\lambda x}{l}$$

λ has units of force (N). Strings only pull; springs push or pull.

Elastic potential energy

Energy stored in a stretched / compressed elastic element:

$$\text{EPE} = \frac{\lambda x^2}{2l}$$

Derived from $\int T\,dx = \int_0^x \frac{\lambda u}{l}\,du$.

Only count EPE when extension $x > 0$.

Other energy forms

Kinetic energy: $KE = \frac{1}{2}mv^2$.

Gravitational PE: $GPE = mgh$ (relative to a reference level).

Pick the reference level for h at the start of the problem and stick with it.

Conservation of energy

Total mechanical energy is conserved when only conservative forces act:

$$\sum E_i = \sum E_f$$

i.e. $KE_i + GPE_i + EPE_i = KE_f + GPE_f + EPE_f$.

Work-energy theorem

For a particle acted on by total force $\mathbf{F}_{\text{net}}$:

$$W_{\text{net}} = \Delta KE$$

For non-conservative forces (friction, resistance):

$$W_{\text{nc}} = \Delta(KE + GPE + EPE)$$

Power

Rate of doing work:

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$$

For a vehicle at speed v with driving force F in line of motion: $P = Fv$.

Watts $=$ joules per second.

Spring at equilibrium

Mass hanging on a vertical spring/string with extension x_0:

$$\frac{\lambda x_0}{l} = mg \Rightarrow x_0 = \frac{mgl}{\lambda}$$

Useful for "released from rest, find max extension" via energy conservation.

Common pitfalls

- Double-counting GPE and EPE — both are stored energies but distinct.
- Forgetting the factor $\frac{1}{2}$ in EPE.
- Including EPE when the string is slack.
- Using $P = Fv$ when there is a non-zero angle between $\mathbf{F}$ and $\mathbf{v}$.

Strategy

1. Identify all forms of energy at start and end positions.
2. Equate using conservation, or include W_{nc} if there's friction.
3. For power: relate to Fv at the instant the speed is given.
4. For greatest extension: solve quadratic in x from energy conservation.

SECTION T2

Hooke's Law, Work, Energy & Power

Questions 1–13 · 109 marks

1. One end of a light elastic string, of natural length 0·8 m, is attached to a fixed point O, and the other end is attached to a particle of mass 5 kg. When the particle hangs in equilibrium vertically below O, the length of the string is 1·3 m.

(a) Calculate the modulus of elasticity of the string. [3]

(b) Determine the elastic energy stored in the string. [2]

1. An elastic string, of natural length 0·3 m, supports a weight of 12 N hanging freely in equilibrium. The total length of the string is 0·55 m.

(a) Calculate the modulus of elasticity of the string. [3]

(b) Find the elastic energy stored in the string. [3]

2. One end of a light elastic string, of natural length $\frac{5}{3}$ m and modulus of elasticity 245 N, is attached to a fixed point *O*. The other end of the string is attached to a particle of mass 7·5 kg. The particle hangs in equilibrium vertically below *O*.

(a) Calculate the extension of the string. [3]

(b) Determine the elastic energy stored in the string. [2]

4. A light elastic string, of natural length 0·8 m and modulus of elasticity 35·4 N, has one end *A* attached to a fixed point and the other end *B* attached to a particle *P* of mass 3 kg. Initially *P* is held at rest at *A*. It is then released and allowed to fall. Calculate the speed of *P* when the length of the string is 1·2 m. [7]

3. The end A of a light elastic string AB, of natural length 0·8 m, is fixed. A particle P, of mass 3 kg, is attached to the end B of the string. Initially, P is held at rest at the point A. It is then released and allowed to fall. The greatest extension of the string in the subsequent motion is 0·4 m.

(a) Show that the modulus of elasticity of the string is 352·8 N. [7]

(b) Find the tension in the string when P is at its lowest point and deduce the magnitude of the acceleration of P in this position. [5]

3. A particle P, of mass 3 kg, is attached to one end A of a light elastic string of natural length 2 m. The other end B of the string is attached to a point on the ceiling. The modulus of elasticity of the string is 294 N.

(a) The particle P is suspended in equilibrium. Calculate the extension of the string AB with A vertically below B. [3]

(b) The particle P is held at a distance of 1·2 m vertically below B and is then released. Determine the speed of P as it passes through the equilibrium position. [8]

5. A light elastic string, of natural length 1·6 m and modulus of elasticity 80 N, has one end attached to a fixed point *A* and the other end attached to a particle *P*, of mass 4 kg. Initially, *P* is held at a point 0·5 m vertically below the point *A*. The particle *P* is released from rest and allowed to fall.

(a) Calculate the tension in the string when the length of the string is 2 m. [2]

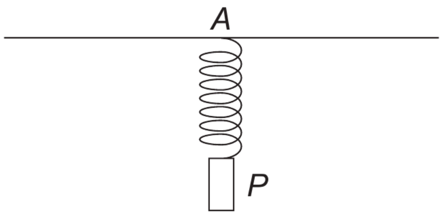
(b) Determine the speed of *P* when the length of the string is 2 m. [8]

7. The end A of a light elastic string AB, of natural length 1·2 m and modulus of elasticity 360 N, is fixed. A particle P, of mass 2 kg, is attached to the end B. Initially, P is held at rest at a point which is 0·7 m vertically below A. It is then released and allowed to fall.

(a) Find the greatest extension of the string in the subsequent motion. Give your answer correct to 2 decimal places. [7]

(b) Calculate the velocity of the particle when it is 1·2 m below A. [4]

5. The diagram shows a light spring of natural length 0·4 m and modulus of elasticity 1470 N with one end A fixed and the other end attached to an object P of mass 15 kg.



Initially, P hangs in equilibrium with the spring vertical.

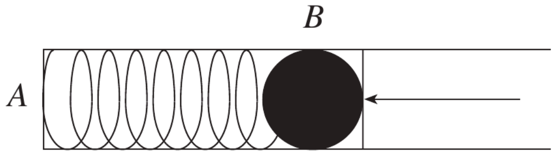
(a) Determine the extension of the spring. [3]

The object P is pulled downwards so that the total length of the spring is 0·56 m. It is then released.

(b) Calculate the speed of P when it is at a distance 0·45 m from A. [8]

5. A particle is attached to one end of a light elastic string of natural length l m and modulus of elasticity λ N. The other end of the string is attached to the ceiling. The particle hangs in equilibrium. The length of the string is 0·95 m when the weight of the particle is 30 N, and 1·15 m when the weight of the particle is 70 N. Find the value of l and the value of λ. [6]

2. The diagram shows a spring of natural length of 0·25 m in a smooth horizontal tube with one end A fixed and a small ball bearing B of mass 0·36 kg held in equilibrium by a force of magnitude 80 N compressing it against the free end of the spring. The length of the compressed spring is 0·2 m.



(a) Find the modulus of elasticity of the spring. [3]

(b) The ball bearing is released by removing the force. Find, by using energy considerations, the speed of the ball bearing just as the spring attains its natural length. [5]

1. The diagram shows a piston, of mass 0·8 kg, enclosed in a horizontal tube and attached to a light spring of natural length 0·2 m and modulus of elasticity 625 N. The other end of the spring is fixed to the end of the tube at point *B*.

Initially, the piston is held at rest at a point *A* with the spring compressed a distance of 0·1 m, so that *AB* is the compressed length of the spring.

(a) Calculate the elastic energy stored in the spring. [2]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude 46 N.

(b) Determine the velocity of the piston when the spring reaches its natural length. [5]

3. A small block, of mass 0·4 kg, lies on a smooth plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$. The block is attached to one end of a light elastic string of natural length 0·7 m and modulus 19·6 N. The other end of the string is attached to a fixed point A. The block is below the level of A and the string is parallel to a line of greatest slope of the plane. Initially, the block is held with the string extended by 0·5 m.

(a) Find the initial tension in the string. [2]

(b) Calculate the initial energy stored in the string. [2]

The block is now released.

(c) Calculate the speed of the block when the string just becomes slack. [6]

END OF HOOKE'S LAW, WORK, ENERGY & POWER PACK

Source: WJEC M2 (2008 modular spec) · 2005–2017

Curated for WJEC FM 2017 spec AS Unit 3 — Topic 2 (2.3.2)