

Name	Date started	Target end date
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## GCE AS / A LEVEL – FURTHER MECHANICS A QUESTION PACK

0981-01 (Legacy M2) & 0982-01 (Legacy M3) · New spec Unit 3 Topic 1

# REVISE

.wales

## FURTHER MATHS – MECH A · MOMENTUM & IMPULSE

### *Momentum, Impulse & Newton's Experimental Law*

*Every momentum / impulse / impulsive-jerk question from the legacy WJEC M2 + M3 papers (June 2005 – June 2017 + Specimen) that maps onto new-spec AS Unit 3 Topic 1.*

#### LEGACY 2008 SPECIFICATION

#### Estimated time for entire question pack: ~3 hours 27 minutes

*Derived from the legacy M2/M3 paper's pace of ~1.5 min/mark (138 marks over 13 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

#### ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every momentum & impulse question from the legacy WJEC M2/M3 papers (2008 modular spec) that maps onto new-spec AS Unit 3 Topic 1 (2.3.1).

Questions are ordered roughly by topic / difficulty.

#### INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

*A calculator is allowed. The WJEC Formula Booklet may be referred to. Take  $g = 9.8 \text{ m s}^{-2}$  unless told otherwise.*

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Q	Source	Max	Mark	Q	Source	Max	Mark
1	Jun 17 Q4	12		8	Jun 10 Q4	11	
2	Jun 16 Q4	9		9	Jun 11 Q5	10	
3	Jun 15 Q4	10		10	Jun 13 Q4	9	
4	Jun 06 Q5	12		11	Jun 15 Q5	11	
5	Jun 07 Q4	14		12	Jun 17 Q2	6	
6	Jun 08 Q6	9		13	Spec. Q5	13	
7	Jun 09 Q4	12					
				<b>Total</b>		<b>138</b>	

# Momentum, Impulse & Newton's Experimental Law – what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 3: Further Mechanics A · Topic 2.3.1.

## Momentum & impulse 2.3.1

- Linear momentum of a particle of mass  $m$  with velocity  $\mathbf{v}$ :  $\mathbf{p} = m\mathbf{v}$ .
- Impulse of a constant force  $\mathbf{F}$  over time  $t$ :  $\mathbf{J} = \mathbf{F}t$ .
- Variable force:  $\mathbf{J} = \int \mathbf{F} dt = \Delta\mathbf{p}$ .
- Impulsive tension when a string suddenly becomes taut – treat as instantaneous.

## Conservation of momentum 2.3.1

- For an isolated system of two particles colliding:  
 $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ .
- Generalises to  $n$  particles:  $\sum m_iu_i = \sum m_iv_i$ .
- Applies to *1D* direct impacts and to *jerk problems* with light strings.
- Momentum is conserved even though KE is generally *not* conserved.

## Newton's Experimental Law (NEL) 2.3.1

- Direct impact of two smooth bodies:  $v_1' - v_2' = -e(u_1 - u_2)$ .
- Coefficient of restitution  $e$  satisfies  $0 \leq e \leq 1$ .
- $e = 1$ : perfectly elastic (KE conserved);  $e = 0$ : perfectly inelastic (particles coalesce).
- Impact with a fixed plane normal: outgoing normal speed =  $e \times$  incoming normal speed.

## Working scientifically general

- Pick a positive direction **before** writing momentum equations – sign errors are the #1 marks lost.
- For 2D impulses on a single particle, split into components.
- KE loss in collision:  $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1v_1^2 + m_2v_2^2)$ .
- For string jerks: the impulsive tension acts equally and oppositely on the two bodies.

# Momentum & Impulse in one page

Quick-reference notes – revisit before each section. Don't use during questions.

## Momentum

Linear momentum of a particle:

$$\mathbf{p} = m\mathbf{v}$$

Units:  $\text{kg m s}^{-1}$  (equivalent to N s).

For a system: total momentum is the vector sum of individual momenta.

## Impulse

Impulse of a constant force  $\mathbf{F}$  over time  $t$ :

$$\mathbf{J} = \mathbf{F}t$$

Variable force:  $\mathbf{J} = \int_0^t \mathbf{F} dt$ .

Impulse-momentum theorem:  $\mathbf{J} = \Delta\mathbf{p} = m\mathbf{v} - m\mathbf{u}$ .

## Conservation of momentum

For an isolated system (no external impulses):

$$\sum m_i \mathbf{u}_i = \sum m_i \mathbf{v}_i$$

Apply *component-wise* in 2D / 3D.

Choose +ve direction before writing the equation.

## Newton's Experimental Law

For direct impact between two smooth bodies:

$$v_1' - v_2' = -e(u_1 - u_2)$$

$0 \leq e \leq 1$  (coefficient of restitution).

$e = 1$ : perfectly elastic (KE conserved).

$e = 0$ : particles stick.

## Impact with a plane

For a ball hitting a smooth fixed plane:

Normal component:  $v_n' = -e u_n$ .

Tangential component: unchanged ( $v_t' = u_t$ ).

Speed after:  $|v'| = \sqrt{e^2 u_n^2 + u_t^2}$ .

## KE lost in a collision

$$\Delta KE = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 - \frac{1}{2}m_1 v_1'^2 - \frac{1}{2}m_2 v_2'^2$$

For an inelastic ( $e < 1$ ) collision,  $\Delta KE > 0$ .

For  $e = 1$ :  $\Delta KE = 0$ .

Express as a fraction of initial KE for "energy lost" parts.

## Impulsive tension in a jerk

When a slack light string suddenly becomes taut, an *impulsive tension*  $\mathbf{J}$  acts equally and oppositely on the two ends.

Along the string:  $m_1(u_1 - v_1) = J = m_2(v_2 - u_2)$ .

If  $B$  is projected at angle  $\theta$  to line  $AB$ , only the component along  $AB$  contributes.

## Common pitfalls

- Sign of velocity after collision – pick +ve once and stick with it.
- Forgetting to scale impulse to component along the string in 2D jerks.
- Using  $e = 1$  when energy is asked to be lost – check the question.
- Treating impulsive tension as a normal tension (it acts instantaneously).

## Strategy

1. Draw the before / after diagram with chosen positive direction.
2. Write momentum conservation along that direction.
3. If  $e$  is given, add NEL:  $v_1' - v_2' = -e(u_1 - u_2)$ .
4. Solve simultaneously. For 2D, split into components.

# SECTION T1

## *Momentum & Impulse*

Questions 1-13 · 138 marks

4.  $A$  and  $B$  are points a distance 18 m apart on horizontal ground. An object  $P$  is projected from  $A$  towards  $B$  with velocity  $15 \text{ ms}^{-1}$  at an angle of  $60^\circ$  to the horizontal. Simultaneously, another object  $Q$  is projected from  $B$  towards  $A$  with velocity  $v \text{ ms}^{-1}$  at an angle of  $30^\circ$  to the horizontal. The objects collide.
- (a) Find the value of  $v$ . [5]
- (b) Show that the time from projection to collision is 0.6 seconds. [3]
- (c) Determine the speed of the object  $P$  just before collision. [4]

4. By burning a charge, a cannon fires a cannon ball of mass 12 kg horizontally. As the cannon ball leaves the cannon, its speed is  $600 \text{ ms}^{-1}$ . The recoiling part of the cannon has a mass of 1600 kg.
- (a) Determine the speed of the recoiling part immediately after the cannon ball leaves the cannon. [3]
- (b) Find the energy created by the burning of the charge. State any assumption you have made in your solution. [4]
- (c) Calculate the constant force needed to bring the recoiling part to rest in 1.2 m. [2]

4. A particle of mass  $0.5 \text{ kg}$  is moving under the action of a single force  $\mathbf{F} \text{ N}$ , where  $\mathbf{F} = (4t - 3)\mathbf{i} + (3t^2 - 5t)\mathbf{j}$ .
- (a) The velocity of the particle at time  $t \text{ s}$  is  $\mathbf{v} \text{ ms}^{-1}$ . When  $t = 0$ ,  $\mathbf{v} = 8\mathbf{i} - 7\mathbf{j}$ .  
Find an expression for  $\mathbf{v}$  in terms of  $t$ . [5]
- (b) When  $t = 3$ , the particle receives an impulse of  $2\mathbf{i} - 9\mathbf{j} \text{ N s}$ . Find the speed of the particle immediately after the impulse. [5]

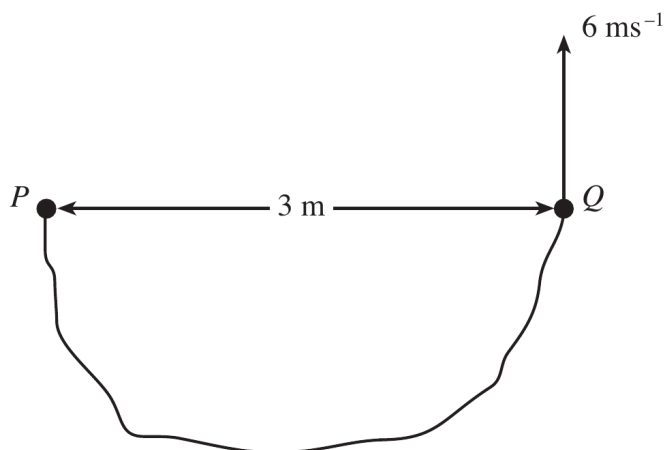
5. Particle  $A$ , of mass 2 kg, and particle  $B$ , of mass 3 kg, are connected by a light inextensible string of length  $l$  m. Initially, both particles are lying at rest on a smooth horizontal surface a distance  $l$  m apart, with the string just slack. Particle  $B$  is given a blow of impulse 40 Ns in a direction away from  $A$  at an angle  $\alpha$  to the line joining the initial positions of  $A$  and  $B$ .



Immediately after the blow, the speed of particle  $A$  is  $4 \text{ ms}^{-1}$ .

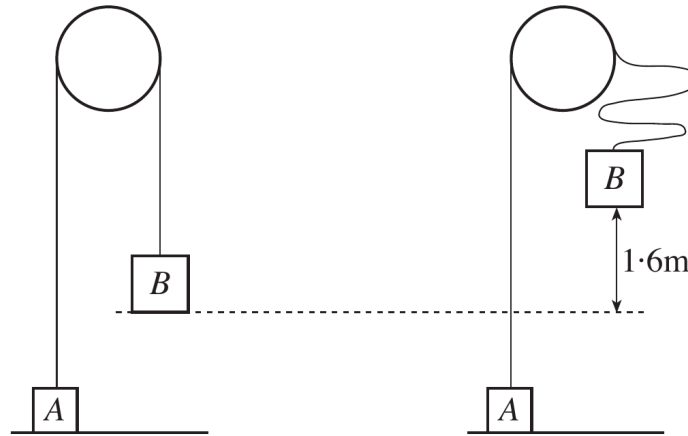
- (a) Determine the value of  $\alpha$ . [6]
- (b) Calculate the magnitude and direction of the velocity of  $B$  immediately after the blow. [6]

4. Two particles  $P$  and  $Q$ , of mass  $7\text{ kg}$  and  $9\text{ kg}$  respectively, are attached one to each end of a light inextensible string of length  $5\text{ m}$ . Initially, the particles are at rest on a smooth horizontal surface a distance  $3\text{ m}$  apart, as shown in the diagram. Particle  $Q$  is then projected horizontally with velocity  $6\text{ ms}^{-1}$  in a direction at  $90^\circ$  to the line joining the initial positions of  $P$  and  $Q$ .



Calculate the speed of  $P$  and the speed of  $Q$  immediately after the string becomes taut. Determine the impulsive tension in the string during the jerk, and find the angle between the velocity of  $P$  and the velocity of  $Q$  immediately after the jerk. [14]

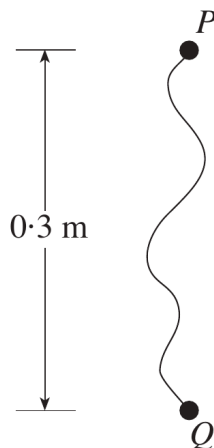
6. A particle  $A$ , of mass  $7\text{ kg}$ , rests on a horizontal table. It is attached to one end of a light inextensible string which passes over a smooth light pulley. The other end of the string is attached to another particle  $B$ , of mass  $3\text{ kg}$ . Initially, the particles are held at rest with the string just taut. Particle  $B$  is raised vertically through a distance of  $1.6\text{ m}$  and released from rest.



Find the speed with which particle  $A$  begins to rise, and the impulsive tension in the string. [9]

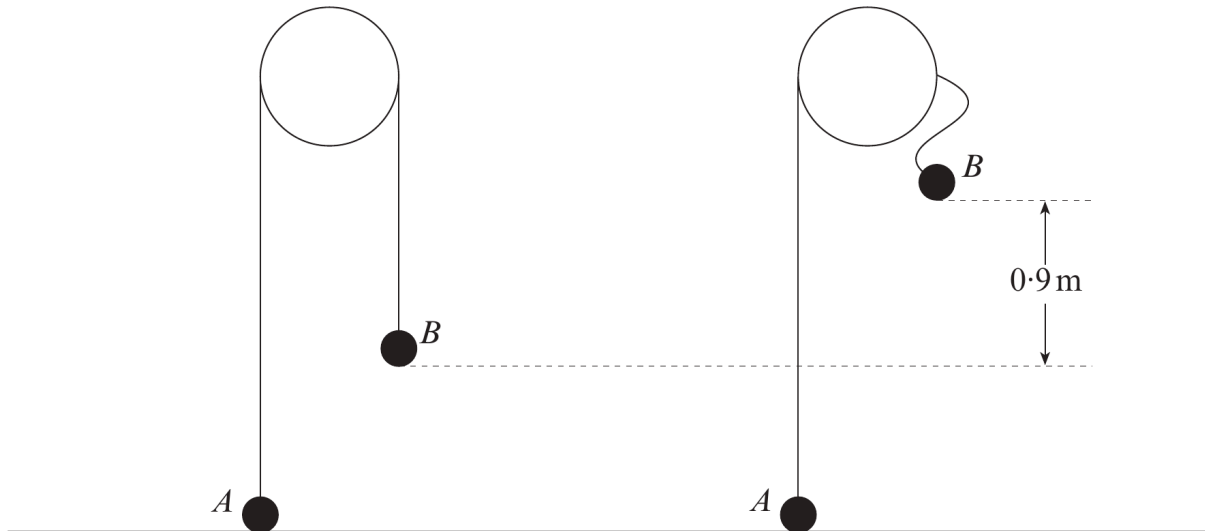
4. Two spheres  $P$  and  $Q$ , of mass 5 kg and 3 kg respectively, rest on a smooth table. They are connected by a light inextensible string which is initially slack. An impulse of magnitude 1.2 Ns is applied to  $Q$  in the direction  $PQ$ .
- (a) Determine the speed with which  $Q$  begins to move. [2]
- (b) Find the speed with which  $P$  moves after the string tightens, and determine the impulsive tension in the string. [6]
- (c) Calculate the loss in energy when the string tightens. [4]

4. Two particles  $P$  and  $Q$ , of mass 3 kg and 5 kg respectively, are attached one to each end of a light inextensible string of length 0.6 m. Initially, the particles are at rest on a smooth horizontal surface a distance 0.3 m apart, as shown in the diagram.



The particle  $Q$  is projected across the surface with speed  $8 \text{ ms}^{-1}$  in a direction at  $90^\circ$  to the line joining the initial positions of  $P$  and  $Q$ . Determine the impulsive tension in the string during the jerk, stating your unit clearly. Find the speed with which each particle begins to move immediately after the jerk. [11]

5. The diagram shows two particles  $A$  and  $B$ , of masses  $4\text{ kg}$  and  $3\text{ kg}$  respectively, connected by a light inextensible string passing over a smooth light pulley fixed above a horizontal plane. Initially, the particle  $A$  is at rest on the plane and particle  $B$  hangs at a depth of  $1.0\text{ m}$  below the pulley.



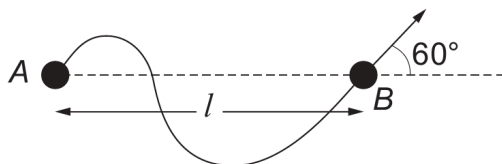
Particle  $B$  is then raised vertically a distance of  $0.9\text{ m}$  and released from rest from that position.

- (a) Calculate the speed of  $B$  immediately before the string tightens. [3]
- (b) Determine the speed with which  $A$  leaves the plane and the impulsive tension in the string immediately after the string tightens. [7]

**TURN OVER**

4. Two particles  $A$  and  $B$ , of masses  $5\text{ kg}$  and  $3\text{ kg}$  respectively, rest on a smooth horizontal surface. Particle  $A$  lies at the edge of the surface and particle  $B$  lies a distance of  $0.2\text{ m}$  from the edge such that the line  $AB$  is perpendicular to the edge of the surface. The two particles are connected by a light inextensible string of length  $1.8\text{ m}$ . Particle  $A$  is then allowed to drop from rest from the edge of the surface. Calculate the speed of  $B$  immediately after the string becomes taut and find the impulsive tension in the string. [9]

5. Two particles  $A$  and  $B$ , of mass  $3\text{ kg}$  and  $5\text{ kg}$  respectively, are attached one to each end of a light inextensible string of length  $\sqrt{3}l\text{ m}$ . Initially, the particles are at rest on a smooth horizontal surface a distance  $l\text{ m}$  apart, as shown in the diagram. Particle  $B$  is then projected horizontally with speed  $8\text{ ms}^{-1}$  at an angle of  $60^\circ$  to the line joining the initial positions of  $A$  and  $B$  produced.



Immediately after the string becomes taut,

- (a) show that the particle  $A$  starts to move in a direction which makes an angle of  $30^\circ$  with the line joining the initial positions of  $A$  and  $B$ . [2]
- (b) find the speed with which each particle begins to move and determine the magnitude of the impulsive tension in the string. [9]

# TURN OVER

2. Two particles  $P$  and  $Q$ , of mass  $3\text{ kg}$  and  $7\text{ kg}$  respectively, are attached one to each end of a light inextensible string. Initially, the string is slack and the particles are at rest on a smooth horizontal surface. The particle  $Q$  is then projected across the surface with speed  $8\text{ ms}^{-1}$  away from  $P$  along the straight line passing through the initial positions of  $P$  and  $Q$ . Find the speed with which the particles begin to move immediately after the jerk and determine the impulsive tension in the string during the jerk. [6]

5. A light inextensible string, of length 1 m, connects particles A and B, of masses 2 kg and 5 kg respectively. Initially, the particles are lying on a smooth horizontal surface 0.6 m apart. Particle B is projected with a velocity of  $7 \text{ ms}^{-1}$  in a direction perpendicular to the line joining the initial positions of A and B.
- (a) Determine the speeds of A and B immediately after the string becomes taut, and find the impulsive tension in the string. [9]
- (b) Calculate the energy lost by the system when the string becomes taut. [4]

## **END OF MOMENTUM & IMPULSE PACK**

Source: WJEC M2/M3 (2008 modular spec) · 2005–2017  
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