Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER STATISTICS A QUESTION PACK

0984-01 (Legacy S2) · New spec Unit 2 Topic 2

REVISE
.wales

FURTHER MATHS — FS A · POISSON DISTRIBUTION

The Poisson Distribution

Every Poisson question from the legacy WJEC S2 papers (June 2005 – June 2017 + Specimen).

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~4 hours 2 minutes

*Derived from the legacy S2 paper's pace of **~1.5 min/mark** (161 marks over 14 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every poisson distribution question from the legacy WJEC S2 papers (2008 modular spec) that maps onto new-spec AS Unit 2 Topic 2 (2.2.1).

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet and statistical tables (Murdoch & Barnes) may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 05 Q2	12	
2	Jun 05 Q4	8	
3	Jun 06 Q4	8	
4	Jun 06 Q5	12	
5	Jun 07 Q6	17	
6	Jun 08 Q7	17	
7	Jun 09 Q1	8	

Q	Source	Max	Mark
8	Jun 10 Q2	12	
9	Jun 11 Q3	11	
10	Jun 12 Q4	10	
11	Jun 13 Q5	13	
12	Jun 14 Q6	13	
13	Jun 17 Q2	7	
14	Spec. Q7	13	
	Total	**161**	

The Poisson Distribution — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 2: Further Statistics A · Topic 2.2.1.

Poisson distribution 2.2.1

- A discrete RV $X \sim Po(\lambda)$ takes values $0, 1, 2, \ldots$ with PMF $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$.
- Mean and variance both equal λ: $E(X) = \lambda$, $\mathrm{Var}(X) = \lambda$.
- Models "rare events" / "rate per unit interval".
- Cumulative probabilities $P(X \leq k)$ available from statistical tables.

Sum of independents 2.2.1

- If $X_1 \sim Po(\lambda_1)$, $X_2 \sim Po(\lambda_2)$ independent, then $X_1 + X_2 \sim Po(\lambda_1 + \lambda_2)$.
- Generalises: $\sum_i X_i \sim Po(\sum_i \lambda_i)$.
- Useful for scaling: rate per minute × T minutes $= \lambda T$.

Poisson approximation to Binomial 2.2.1

- If $X \sim B(n, p)$ with n large ($n \geq 50$) and p small ($np < 5$), then $X \approx Po(np)$.
- Good for rare events in large trial counts.
- Use when binomial tables don't cover the value of n.

Working scientifically general

- Identify the time/space interval and the mean rate — scale λ accordingly.
- Use $1 - P(X \leq k - 1)$ for $P(X \geq k)$ — tables list $\leq$.
- Hypothesis testing: compare observed counts against $Po(\lambda_0)$ tail probabilities.
- Watch for "at least", "more than", "exactly" wording — convert to $\leq$ / $\geq$ / $=$ events carefully.

Poisson Distribution in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Poisson PMF

For $X \sim Po(\lambda)$:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots$$

$E(X) = \mathrm{Var}(X) = \lambda$.

Cumulative probabilities

Use Poisson tables for $P(X \leq k)$.

$P(X \geq k) = 1 - P(X \leq k - 1)$

$P(X = k) = P(X \leq k) - P(X \leq k - 1)$

Sum of independents

If $X_1 \sim Po(\lambda_1)$ and $X_2 \sim Po(\lambda_2)$ are independent:

$X_1 + X_2 \sim Po(\lambda_1 + \lambda_2)$

Generalises to any number of independents.

Rate scaling

If events occur at rate λ per unit time, then number in T units is $Po(\lambda T)$.

E.g. "2 calls/hour" $\Rightarrow$ in 3 hours: $Po(6)$.

Combine with rate addition: $\lambda_A + \lambda_B$ for two independent sources.

Approximation to Binomial

If $X \sim B(n, p)$ with n large, p small:

$$X \approx Po(np)$$

Rule of thumb: $n \geq 50, np < 5$. Useful when n exceeds binomial tables.

Hypothesis testing

Given $X \sim Po(\lambda_0)$ under H_0:

One-tailed test: compute $P(X \geq x_{\text{obs}})$ or $P(X \leq x_{\text{obs}})$.

Compare to significance level α — reject if tail probability is smaller.

Mean & variance relation

$E(X^2) = \mathrm{Var}(X) + [E(X)]^2 = \lambda + \lambda^2$.

Used when given $E(X^2)$ and asked to find λ.

Solve $\lambda^2 + \lambda - E(X^2) = 0$ via the quadratic formula.

Common pitfalls

- Forgetting to scale λ for non-unit intervals.
- Using $P(X < k)$ instead of $P(X \leq k - 1)$ — identical for discrete X but tables list $\leq$.
- Adding rates that aren't independent.
- Using Poisson approximation when p is not small.

Strategy

1. Identify the rate λ for the specific interval/region.
2. Translate the event into $X \leq k, \geq k$, or $= k$.
3. Look up $P(X \leq k)$ in Poisson tables.
4. For tests: compute exact tail probability and compare to α.

SECTION T2

Poisson Distribution

Questions 1–14 · 161 marks

2. The number of batteries sold per week by a garage may be assumed to have a Poisson distribution with a mean 5.

(a) Find the probability that

(i) exactly 6 are sold in a randomly chosen week, [2]
(ii) exactly 6 are sold in each of 3 randomly chosen weeks, [2]
(iii) exactly 18 are sold in a randomly chosen 3-week period. [3]

(b) Find, approximately, the probability that more than 240 are sold in a randomly chosen 52-week period. [5]

4. The number of machine breakdowns per day in a certain factory may be assumed to have a Poisson distribution with mean μ. The value of μ is known, from past experience, to be 1·5. In an attempt to reduce the value of μ, all the machines are fitted with new control units. To investigate whether or not this succeeds in reducing the value of μ, the number of breakdowns, x, during a 30-day period following the fitting of these new units is recorded.

(a) State suitable hypotheses. [1]

(b) It is decided to conclude that the value of μ has been reduced if $x \leqslant 35$.

(i) What name is given to the region $x \leqslant 35$?

(ii) Define the term 'significance level' and find its approximate value for this procedure. [7]

4. There are 5 computers in an office working continuously. You may assume that, for each computer independently of the others, the number of 'crashes' occurring during a week follows a Poisson distribution with mean 0·8.
During a randomly chosen week, find the probability that

(a) each computer crashes exactly once, [4]

(b) the total number of crashes on all the computers is five. [4]

5. On any weekday, the number of passengers, X, using an early morning bus service may be assumed to follow a Poisson distribution. In the past, the mean value of X has been 2·4. The local council wishes to increase this mean so they decide to offer this service free of charge.

(a) During the 5-day week following this offer, a total of 18 passengers used this service. Clearly stating your hypotheses, calculate the p-value of this result. Interpret your value in context. [5]

(b) During the 100 weekdays following this offer, a total of 280 passengers used this service. Determine, at the 1% significance level, whether or not the offer has resulted in an increase in the mean number of passengers using this early morning bus service. [7]

6. A plumber knows that the number of emergency calls received per day follows a Poisson distribution with mean $\mu = 2$.

(a) Calculate the probability that, in a 7-day period, he receives

(i) exactly 10 calls,

(ii) more than 12 calls. [5]

(b) Wishing to increase the value of μ, he increases his advertising budget.

(i) State suitable hypotheses for investigating whether or not this achieves the desired result.

(ii) In the first 7-day period after increasing the budget, he receives 20 emergency calls. Calculate and interpret the p-value of this result.

(iii) In the next 100-day period, he receives 230 emergency calls. Calculate an approximate p-value of this result and interpret it. [12]

7. The random variable X has a Poisson distribution with unknown mean μ. It is required to test whether $\mu = 2{\cdot}5$ against a two-sided alternative.

(a) State suitable hypotheses. [1]

(b) Let S denote the sum of six randomly chosen values of X. It is decided to reject the null hypothesis if either $S \leqslant 8$ or $S \geqslant 23$.

(i) Calculate the significance level of this test.

(ii) Given that the value of μ is actually 2, find the probability of reaching an incorrect conclusion. [8]

(c) It is now decided to obtain a random sample of 100 values of X. It is found that the sum of these 100 values is 270. Find an approximate p-value of this sum and interpret your result. [8]

1. The number of telephone enquiries received per hour at a certain office may be assumed to follow a Poisson distribution with mean μ. Office records indicate that $\mu = 2$ but the office manager believes that the value of μ has increased.

(a) To test this belief, he counts the number of enquiries received during a 6-hour period. Given that 18 enquiries are received, calculate the p-value. [3]

(b) He now decides to count the number of enquiries received during a 50-hour period. Given that 125 enquiries are received, calculate the p-value and state your conclusion. [5]

2. The number of computer breakdowns per day at a large office may be assumed to follow a Poisson distribution with mean μ. The IT Manager believes that the value of μ should be 1·5 but he decides to check this. He therefore defines the following hypotheses.

$$H_0{:}\mu = 1{\cdot}5; \quad H_1{:}\mu \neq 1{\cdot}5$$

(a) For one test, he decides to count the number of breakdowns, x, in a 10-day period and to define the critical region as $x \leqslant 9$ or $x \geqslant 22$. Find the significance level of this test. [5]

(b) For another test, he decides to count the number of breakdowns occurring during a 100-day period. Given that 170 breakdowns occur, calculate the approximate p-value and state your conclusion. [7]

3. A factory manufactures screws and packs them in large bags. The number of defective screws in a bag can be modelled by a Poisson distribution whose mean is known to have been 0·5. However, new equipment has been installed which, it is hoped, will decrease this mean. The Quality Controller plans to take samples of bags to investigate whether or not there is a reduction in the mean.

(a) State suitable hypotheses. [1]

(b) He takes a random sample of 30 bags and finds that they contain a total of 12 defective screws. Calculate the p-value and state your conclusion. [4]

(c) He then takes a random sample of 200 bags and finds that they contain a total of 80 defective screws. Calculate an approximate p-value and state your conclusion. [6]

4. *(a)* When Jack types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean 0·8. He types a 10-page document. Determine the probability that the total number of errors is less than 5. [3]

(b) When Mary types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean μ. Mary claims that the value of μ is less than 0·8 but Jack claims that μ is equal to 0·8.

(i) State suitable hypotheses for testing these claims.

(ii) Mary types an 80-page document and makes 60 errors. Find the approximate p-value of this result and state your conclusion. [7]

5. The number of accidents occurring per day along a certain stretch of road can be modelled by a Poisson distribution. The value of the mean μ has been 1·2 in the past but the local council has recently introduced a lower speed limit in the hope of reducing the value of μ.

(a) State suitable hypotheses for testing whether or not lowering the speed limit has had the desired effect. [1]

(b) It is decided to count the number of accidents, x, occurring in a 60-day period and to define the critical region as $x \leqslant 58$.

(i) Determine the significance level.

(ii) Given that the value of μ has actually fallen to 0·8, determine the probability of concluding that there has been no reduction in the value of μ. [12]

6. When John types a page of a document, the number of errors can be modelled by a Poisson distribution with mean μ. He claims that the value of μ is 1·5 but his employer wants to test this claim so they define the following hypotheses.

$$H_0 : \mu = 1{\cdot}5; \quad H_1 : \mu \neq 1{\cdot}5$$

(a) John is asked to type a 10-page document and the critical region is taken as $x \leqslant 10$ or $x \geqslant 20$, where x denotes the total number of errors in the document.

(i) Find the significance level of this test.

(ii) Find the probability of incorrectly accepting H_0 when the value of μ is actually 1·0. [7]

(b) John now types a 50-page document and makes 92 errors. Find the *p*-value and state your conclusion. [6]

2. The number of computer breakdowns per day in a large IT Department may be assumed to follow a Poisson distribution with mean 0·8. In an attempt to reduce the number of breakdowns, the IT Manager moves the department to a new purpose-built office. He defines the following hypotheses

$$H_0: \mu = 0{\cdot}8\ ; \quad H_1: \mu < 0{\cdot}8$$

where μ denotes the mean number of breakdowns per day after the move.
He finds that in the first 100 days after the move, there was a total of 64 computer breakdowns. You may assume that the numbers of breakdowns on successive days are independent. Calculate the approximate *p*-value of this result and interpret it in context. [7]

Poisson distribution. In the past, the mean value of X has been 3. The manager wishes to increase this mean and she reduces the price in the hope of doing that.

(a) During the following 5-day week, the shop sells 20 video recorders.

(i) State the hypotheses required to determine whether or not the mean has increased.

(ii) Calculate the p-value of this result and interpret it. [5]

(b) In the longer term, the shop sells 330 video recorders in 100 days.

(i) Calculate the p-value of this result.

(ii) State, with a reason, whether or not the result is significant at the 5% level. [8]

END OF POISSON DISTRIBUTION PACK

Source: WJEC S2 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 2 — Topic 2 (2.2.1)