

Name	Date started	Target end date
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GCE AS / A LEVEL – FURTHER STATISTICS A QUESTION PACK

0984-01 (Legacy S2) & 0985-01 (Legacy S3) · New spec Unit 2 Topic 1

REVISE

.wales

FURTHER MATHS – FS A · RANDOM VARIABLES & LINEAR COMBINATIONS

Mean, Variance & Linear Combinations of Random Variables

Every linear-combinations question from the legacy WJEC S2 / S3 papers (June 2005 – June 2017) mapping onto new-spec AS Unit 2.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 38 minutes

Derived from the legacy S2/S3 paper's pace of ~1.5 min/mark (105 marks over 11 questions).

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every random variables & linear combinations question from the legacy WJEC S2/S3 papers (2008 modular spec) that maps onto new-spec AS Unit 2 Topic 1 (2.2.1).

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet and statistical tables (Murdoch & Barnes) may be referred to.

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Q	Source	Max	Mark
1	Jun 12 Q1	8	
2	Jun 07 Q2	9	
3	Jun 08 Q2	9	
4	Jun 09 Q5	12	
5	Jun 10 Q4	14	
6	Jun 14 Q5	8	
7	Jun 17 Q1	13	
8	Jun 06 Q1	8	
9	Jun 07 Q1	8	
10	Jun 08 Q1	10	
11	Jun 17 Q2	6	
Total		105	

Mean, Variance & Linear Combinations of Random Variables

– what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 2: Further Statistics A · Topic 2.2.1.

Discrete random variables 2.2.1

- Define a discrete random variable X with values x_i and probabilities $p_i = P(X = x_i)$, where $\sum p_i = 1$.
- Compute mean $E(X) = \sum x_i p_i$ and variance $\text{Var}(X) = E(X^2) - [E(X)]^2$.
- Compute expectation of a function $E[g(X)] = \sum g(x_i) p_i$ – e.g. $E(X^2)$.
- Find sampling distributions when sampling without replacement from a finite list.

Linear combinations 2.2.1

- $E(aX + b) = a E(X) + b$
- $\text{Var}(aX + b) = a^2 \text{Var}(X)$
- For independent X, Y : $E(aX + bY) = a E(X) + b E(Y)$
- For independent X, Y : $\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$
- For independent X, Y : $E(XY) = E(X) E(Y)$

Common slip-ups general

- Forgetting the b^2 in $\text{Var}(aX + bY)$ – both coefficients are squared.
- Using $\text{Var}(X - Y) = \text{Var}(X) - \text{Var}(Y)$ – **wrong**. Variances always add for independent RVs.
- Applying $E(XY) = E(X)E(Y)$ when X and Y are *not* independent.
- Forgetting $\text{Var}(X) = E(X^2) - [E(X)]^2$ when given only $E(X^2)$.

Working scientifically general

- Tabulate $x_i, p_i, x_i p_i, x_i^2 p_i$ – the columns directly give $E(X)$ and $E(X^2)$.
- For sample-without-replacement problems, list all combinations systematically.
- Verify $\sum p_i = 1$ before computing moments – catches table errors.
- For independent products $U = XY$, use $E(U) = E(X)E(Y)$ and $\text{Var}(U) = E(X^2)E(Y^2) - [E(X)E(Y)]^2$.

Random Variables & Linear Combinations in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Discrete RV moments

For X taking values x_i with probability p_i :

$$E(X) = \sum x_i p_i$$

$$E(X^2) = \sum x_i^2 p_i$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$E[g(X)]$ for any g

$$E[g(X)] = \sum g(x_i) p_i \text{ for discrete } X.$$

For continuous X : $E[g(X)] =$

$$\int g(x) f(x) dx.$$

Special cases: $g(X) = X, X^2, e^{tX}, \dots$

Linear transform $aX + b$

$$E(aX + b) = a E(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

Constant b shifts the mean but not the variance.

Independent sums

For independent X, Y :

$$E(aX + bY) = a E(X) + b E(Y)$$

$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

Note: b^2 , not b – and variance still adds for differences.

Independent products

For independent X, Y :

$$E(XY) = E(X) E(Y)$$

$$\text{Var}(XY) = E(X^2) E(Y^2) -$$

$$[E(X) E(Y)]^2$$

Derive from $\text{Var}(U) = E(U^2) - [E(U)]^2$ with $U = XY$.

Standard distributions

$$X \sim B(n, p): E(X) = np, \text{Var}(X) = np(1 - p).$$

$$X \sim Po(\lambda): E(X) = \lambda, \text{Var}(X) = \lambda.$$

$E(X^2) = \text{Var}(X) + [E(X)]^2$ – useful for "given $E(X^2)$ " problems.

Sampling without replacement

List all distinct samples of size k from a population of size N .

Probability of each sample: $1/\binom{N}{k}$ (or weighted if some items are identical).

Sample mean / median / sum is itself a discrete RV – tabulate its PMF.

Common pitfalls

- $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$ for independents (NOT minus).
- $E(XY) \neq E(X)E(Y)$ when X, Y are dependent.
- Forgetting to square coefficients in variance.
- Computing $E(X)^2$ instead of $E(X^2)$.

Strategy

1. Identify the distribution(s) and whether independent.
2. Tabulate x, p, xp, x^2p if discrete.
3. Compute E, Var via the formula.
4. For combinations: apply linearity / product rules.

SECTION T1

Random Variables & Linear Combinations

Questions 1-11 · 105 marks

-
1. The random variables X and Y are independent, X with mean 5 and variance 2 and Y with mean 6 and variance 3.
- (a) Determine the values of $E(X^2)$ and $E(Y^2)$. [3]
- (b) Given that $U = XY$, find the mean and variance of U . [5]

2. The independent random variables X and Y are Poisson distributed with means 2 and 3 respectively.

(a) (i) Show that $E(X^2) = 6$ and evaluate $E(Y^2)$.

(ii) Deduce the value of $E(X^2Y^2)$. [5]

(b) The random variable U is defined by

$$U = XY.$$

Determine the standard deviation of U . [4]

2. The random variable X has the binomial distribution $B(10, 0.4)$ and, independently, the random variable Y has the binomial distribution $B(30, 0.3)$.

(a) Show that $E(X^2) = 18.4$ and evaluate $E(Y^2)$. [5]

(b) The random variable U is defined by

$$U = XY.$$

Determine the mean and variance of U . [4]

5. (a) The random variable X has the binomial distribution $B(20, 0.4)$. Find the value of
- (i) $E(X)$,
 - (ii) $E(X^2)$. [4]
- (b) The random variable Y has a Poisson distribution with mean μ . Given that $E(Y^2) = 9.36$, determine the value of μ . [4]
- (c) The random variables X and Y are independent and $U = XY$. Determine the variance of U . [4]

4. (a) The random variable X has the binomial distribution $B(n, p)$. Given that $E(X) = 3$ and $E(X^2) = 11.1$, find the values of n and p . [6]
- (b) The independent random variable Y has the binomial distribution $B(15, 0.4)$ and $U = XY$. Find the mean and variance of U . [8]

5. The random variables X and Y are independent observations from the binomial distribution $B(6, 0.2)$. Given that $U = XY$, determine the value of
- (a) $E(U)$, [2]
- (b) $\text{Var}(U)$. [6]

1. The independent random variables X , Y are distributed such that X is $B(5, 0.4)$ and Y is $B(8, 0.2)$. Given that the random variable W is defined by $W = XY$, determine
- (a) the mean and the variance of W , [9]
- (b) $P(W = 0)$. [4]

1. A random sample of 3 numbers is chosen without replacement from the 5 numbers (1, 1, 2, 3, 4).
- (a) List all the possible samples. [2]
- (b) Determine the sampling distributions of
- (i) the sample mean,
 - (ii) the sample median. [6]

1. Six cards are numbered 1, 2, 2, 3, 3, 3 respectively. Two of these cards are chosen at random without replacement. Let X denote the sum of the two numbers on the chosen cards.

(a) Show that

$$P(X = 3) = \frac{2}{15}. \quad [2]$$

(b) Find the sampling distribution of X . [6]

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1. A bag contains six £2 coins and four £1 coins. A random sample of three coins is selected from the bag without replacement.
- (a) Find the sampling distribution of the total value of these three coins. [7]
- (b) Verify that the expected value of this total is three times the mean value of the ten coins in the bag. [3]

2. Each of three fair dice has its six faces numbered 1, 2, 3, 4, 5, 6 respectively. The three dice are thrown simultaneously and the score on each dice is defined as the number on the uppermost face. Let X denote the highest score on these three dice.

(a) Show that

$$P(X \leq x) = \left(\frac{x}{6}\right)^3 \quad \text{for } x = 1, 2, 3, 4, 5, 6. \quad [2]$$

(b) Deduce an expression in terms of x for $P(X = x)$, valid for $x = 1, 2, 3, 4, 5, 6$. [2]

(c) Determine the most likely value of X . [2]

END OF RANDOM VARIABLES & LINEAR COMBINATIONS PACK

Source: WJEC S2/S3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 2 – Topic 1 (2.2.1)

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