Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER PURE MATHEMATICS A QUESTION PACK

2305-01 (Legacy FP1) · New spec Unit 1 Topic 7 (2.1.4)

REVISE
.wales

FURTHER MATHS — FP A · Standard Series Sums & Method of Differences

Summation of Finite Series — Standard Σ-Formulae & Method of Differences

Every standard-series & method-of-differences question from legacy WJEC FP1 (2009–2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~46m

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (31 marks over 5 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, narrowly focused on one sub-topic from Unit 1 (2.1.4). It contains every relevant question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto this sub-topic of new-spec AS Unit 1.

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

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Q	Source	Max	Mark
1	Jun 09 Q1	6	
2	Jan 14 Q2	6	
3	Jun 13 Q1	6	
4	Jun 17 Q2	6	
5	Jun 10 Q6	7	
	Total	31	

Summation of Finite Series — Standard Σ-Formulae & Method of Differences — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 1: Further Pure Mathematics A · Topic 2.1.4.

Standard Σ-formulae 2.1.4

- $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$.
- $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$.
- $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2 = (\sum r)^2$.
- In the formula booklet — quotable directly.

Sigma linearity 2.1.4

- $\sum(au_r + bv_r) = a\sum u_r + b\sum v_r$.
- Use to break a polynomial summand into standard sums.
- e.g. $\sum r(r+1) = \sum r^2 + \sum r$.
- For $\sum r(r+1)^2$: expand to $\sum(r^3 + 2r^2 + r)$.

Method of differences 2.1.4

- If summand can be written as $f(r) - f(r+1)$, the series *telescopes*.
- $\sum_{r=1}^{n}[f(r) - f(r+1)] = f(1) - f(n+1)$.
- Most useful when the summand has a partial-fraction decomposition.
- e.g. $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$.

Partial fractions for telescoping 2.1.4

- $\frac{1}{r(r+2)} = \frac{1}{2}\left(\frac{1}{r} - \frac{1}{r+2}\right)$ — pairs cancel two steps apart.
- $\frac{1}{r(r+1)(r+2)} = \frac{1}{2}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$.
- Sanity check: try $n = 1, 2, 3$ in your closed-form answer.

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Standard Series Sums & Method of Differences in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Standard sums

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$
$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$
$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Sigma linearity

$\sum(au_r + bv_r) = a\sum u_r + b\sum v_r$.

$\sum c = nc$ (constant summed n times).

Break polynomial summands into standard sums.

Worked: $\sum r(r+1)$

$\sum r(r+1) = \sum r^2 + \sum r$

$= \tfrac{1}{6}n(n+1)(2n+1) + \tfrac{1}{2}n(n+1)$

$= \tfrac{1}{3}n(n+1)(n+2)$.

Worked: $\sum r(r+1)^2$

Expand: $r(r+1)^2 = r^3 + 2r^2 + r$.

$\sum = \tfrac{1}{4}n^2(n+1)^2 + \tfrac{1}{3}n(n+1)(2n+1) + \tfrac{1}{2}n(n+1)$.

Factor $\tfrac{1}{12}n(n+1) \Rightarrow \tfrac{1}{12}n(n+1)(3n^2+11n+10)$.

Method of differences

If $u_r = f(r) - f(r+1)$:

$$\sum_{r=1}^{n} u_r = f(1) - f(n+1).$$

Series telescopes — intermediate terms cancel.

Partial fractions for telescoping

$$\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}.$$
$$\frac{1}{r(r+2)} = \frac{1}{2}\left(\frac{1}{r} - \frac{1}{r+2}\right).$$
$$\frac{1}{(2r-1)(2r+1)} = \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right).$$

Telescoping worked example

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \sum\left(\frac{1}{r} - \frac{1}{r+1}\right)$$
$$= \left(1 - \tfrac{1}{2}\right) + \cdots + \left(\tfrac{1}{n} - \tfrac{1}{n+1}\right)$$
$$= 1 - \frac{1}{n+1} = \frac{n}{n+1}.$$

Common pitfalls

- Forgetting to factor the final result.
- Off-by-one in telescoping.
- Algebra slip in partial fractions.

Strategy

1. Expand the summand if polynomial.
2. Standard sums via linearity.
3. Rational summand: partial fractions + telescope.
4. Factor the result.

SECTION D

Standard Series Sums & Method of Differences

Questions 1–5 · 31 marks

1. Given that

$$S_n = \sum_{r=1}^{n} r\left(r+1\right)^2 ,$$

find an expression for S_n in terms of n, giving your answer as a product of linear factors. [6]

2. Given that

$$S_n = 1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \cdots + n(n+1)^2,$$

obtain an expression for S_n, giving your answer as a product of linear factors. [6]

1. Given that

$$S_n = 1^2 + 3^2 + 5^2 + \ldots + (2n - 1)^2,$$

obtain an expression for S_n in the form $an^3 - bn$, where a, b are positive rational numbers. [6]

2. Consider the series

$$S_n = 1^2 + 4^2 + 7^2 + \dots + (3n - 2)^2.$$

Obtain an expression for S_n, giving your answer in the form $an^3 + bn^2 + cn$, where a, b, c are rational numbers. [6]

6. *(a)* Express $\dfrac{1}{r(r+2)}$ in partial fractions. [3]

(b) Hence show that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{3}{4} - \frac{(2n+3)}{2(n+1)(n+2)}\ .$$ [4]

END OF STANDARD SERIES SUMS & METHOD OF DIFFERENCES PACK

Source: WJEC FP1 (2008 modular spec) · 2005–2017

Curated for WJEC FM 2017 spec AS Unit 1 — Topic 7 (2.1.4)

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