| Name | Date started | Target end date |
|---|---|---|
| | | |

GCE AS / A LEVEL — FURTHER PURE MATHEMATICS A QUESTION PACK

2305-01 (Legacy FP1) · New spec Unit 1 Topic 6 (2.1.4)

REVISE
.wales

# FURTHER MATHS — FP A · Roots of Polynomials

## *Roots of Polynomials — Vieta's Relations & Symmetric Sums*

*Every roots-of-polynomials question from legacy WJEC FP1 (2010–2015)*

LEGACY 2008 SPECIFICATION

### Estimated time for entire question pack: ~1h 10m

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (47 marks over 6 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

### ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, narrowly focused on one sub-topic from Unit 1 (2.1.4). It contains every relevant question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto this sub-topic of new-spec AS Unit 1.

Questions are ordered roughly by difficulty.

### INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

*A calculator is allowed. The WJEC Formula Booklet may be referred to.*



*For Examiner's use only*

| Q | Source | Max | Mark |
|---|---|---|---|
| 1 | Jun 14 Q5 | 10 | |
| 2 | Jun 13 Q3 | 11 | |
| 3 | Jun 10 Q4 | 8 | |

| Q | Source | Max | Mark |
|---|---|---|---|
| 4 | Jan 10 Q5 | 6 | |
| 5 | Jun 15 Q5 | 5 | |
| 6 | Jun 11 Q9 | 7 | |
| | **Total** | **47** | |

# Roots of Polynomials — Vieta's Relations & Symmetric Sums — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 1: Further Pure Mathematics A · Topic 2.1.4.

### Vieta — quadratic 2.1.4

- For $ax^2+bx+c=0$ with roots $\alpha, \beta$:
- $\alpha+\beta=-\frac{b}{a}, \quad \alpha\beta=\frac{c}{a}$.

### Vieta — cubic 2.1.4

- For $ax^3+bx^2+cx+d=0$ with roots $\alpha, \beta, \gamma$:
- $\sum \alpha=-\frac{b}{a}$
- $\sum \alpha\beta=\frac{c}{a}$
- $\alpha\beta\gamma=-\frac{d}{a}$.

### Symmetric sums 2.1.4

- $\sum \alpha^2=(\sum \alpha)^2-2\sum \alpha\beta$.
- $\sum \alpha^3=(\sum \alpha)^3-3(\sum \alpha)(\sum \alpha\beta)+3\alpha\beta\gamma$ (cubic).
- $\sum \frac{1}{\alpha}=\frac{\sum \alpha\beta}{\alpha\beta\gamma}$.

### New equations with related roots 2.1.4

- Find equation with roots $\alpha+k$: substitute $x=y-k$ into the original.
- Find equation with roots $\alpha^2$: substitute $x=\sqrt{y}$, isolate radical, square.
- Find equation with roots $\frac{1}{\alpha}$: substitute $x=\frac{1}{y}$, multiply by $y^n$.

# Roots of Polynomials in one page

Quick-reference notes — revisit before each section. Don't use during questions.

### Quadratic Vieta

$ax^2 + bx + c = 0$, roots $\alpha, \beta$:

$\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

### Cubic Vieta

$ax^3 + bx^2 + cx + d = 0$, roots $\alpha, \beta, \gamma$:

$\sum \alpha = -\frac{b}{a}$

$\sum \alpha\beta = \frac{c}{a}$

$\alpha\beta\gamma = -\frac{d}{a}$

### Quartic Vieta

$ax^4 + \ldots = 0$, roots $\alpha, \beta, \gamma, \delta$:

$\sum \alpha = -b/a$, $\sum \alpha\beta = c/a$, $\sum \alpha\beta\gamma = -d/a$, $\alpha\beta\gamma\delta = e/a$.

### Power sums

$\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$.

$\sum \alpha^3 = (\sum \alpha)^3 - 3(\sum \alpha)(\sum \alpha\beta) + 3\alpha\beta\gamma$ (cubic).

$\sum \frac{1}{\alpha} = \frac{\sum \alpha\beta}{\alpha\beta\gamma}$.

### Symmetric expressions

For cubic with roots $\alpha, \beta, \gamma$:

$\beta\gamma + \gamma\alpha + \alpha\beta = \sum \alpha\beta = \frac{c}{a}$.

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -\frac{c}{d}$.

### Equation with new roots

For roots $\alpha + k$: substitute $x = y - k$.

For roots $\alpha^2$: substitute $x = \sqrt{y}$, square.

For roots $1/\alpha$: substitute $x = 1/y$, multiply by $y^n$.

### GP roots (geometric)

If three roots are in GP, write them as $\frac{\alpha}{r}, \alpha, \alpha r$.

Product: $\alpha^3 = \alpha\beta\gamma = -\frac{d}{a}$.

So $\alpha = \sqrt[3]{-d/a}$.

### Equal-roots condition

If cubic has repeated root:

Derivative shares a root with the original.

e.g. $x^3 + qx + r = 0$ with equal roots $\Leftrightarrow 4q^3 + 27r^2 = 0$.

### Strategy

**1.** Write Vieta relations.
**2.** Express target sum/product via Vieta.
**3.** Substitute to get answer.

# SECTION D

## *Roots of Polynomials*

Questions 1–6 · 47 marks

**5.** The roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0$$

are denoted by $\alpha$, $\beta$, $\gamma$.

*(a)* Find the cubic equation whose roots are $\beta\gamma$, $\gamma\alpha$, $\alpha\beta$. [6]

*(b)* Show that

$$\alpha^2 + \beta^2 + \gamma^2 = 0.$$

Deduce the number of real roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0,$$

justifying your answer. [4]

**3.** The roots of the cubic equation $x^3 - 2x^2 + 2x + 1 = 0$ are denoted by $\alpha, \beta, \gamma$ .

*(a)* Show that

$$\frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} + \frac{\alpha\beta}{\gamma} = -8.$$ [4]

*(b)* Find the cubic equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$. [7]

**4.** The roots of the quadratic equation

$$x^2 + 2x + 3 = 0$$

are denoted by $\alpha$, $\beta$. Find the quadratic equation whose roots are

$$\alpha - \frac{1}{\beta^2},\ \beta - \frac{1}{\alpha^2}.$$ [8]

**5.** Given that the cubic equation $x^3 - qx + r = 0$ has two equal roots, show that

$$4q^3 = 27r^2.$$ [6]

**5.** The roots of the cubic equation

$$x^3 - 4x^2 - 8x + k = 0$$

are in geometric progression. Determine the value of $k$. [5]

**9.** The roots of the following cubic equation are in geometric progression.

$$x^3 + fx^2 + gx + h = 0$$

Show that $g^3 = f^3h$. [7]

**END OF ROOTS OF POLYNOMIALS PACK**

Source: WJEC FP1 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 1 — Topic 6 (2.1.4)