Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER PURE MATHEMATICS A QUESTION PACK

2305-01 (Legacy FP1) · New spec Unit 1 Topic 5 (2.1.3)

REVISE
.wales

FURTHER MATHS — FP A · Matrices — 2D Linear Transformations

Matrices — 2D Linear Transformations & Composites

Every 2D transformation matrix question from legacy WJEC FP1 (sparse — only 2 in the corpus)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~24m

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (16 marks over 2 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, narrowly focused on one sub-topic from Unit 1 (2.1.3). It contains every relevant question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto this sub-topic of new-spec AS Unit 1.

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 17 Q4	8	

Q	Source	Max	Mark
2	Jan 11 Q8	8	
Total		**16**	

Matrices — 2D Linear Transformations & Composites — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 1: Further Pure Mathematics A · Topic 2.1.3.

2D transformation matrices 2.1.3

- A point (x, y) maps to $M \begin{pmatrix} x \\ y \end{pmatrix}$.
- Rotation by θ (anticlockwise): $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.
- Reflection in line at angle θ to x-axis: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$.
- Enlargement scale k: kI.

Composite transformations 2.1.3

- Apply T_1 then T_2: write T_2T_1 (right-to-left).
- Matrix multiplication is *not* commutative — order matters.
- For three transformations $T_3 \circ T_2 \circ T_1$: write $M_3M_2M_1$.

Translations (homogeneous coords) 2.1.3

- Pure linear maps fix the origin. Translations need **homogeneous coordinates**: $(x, y, 1)^\top$.
- Translation by (a, b): $\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$.
- Combine rotations, reflections, translations as 3×3 matrices.

Corpus note general

- FP1 had very sparse transformation practice — only 2 questions across the whole 2005–2017 corpus.
- For more practice, work through textbook exercises and the 2017 spec sample assessment material.
- Invariant lines / invariant points (also in 2.1.3) had no FP1 questions — spec-only topic.

Crib sheet »

Matrices — 2D Linear Transformations in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Rotation matrix

Anticlockwise rotation by θ:

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

$R_\alpha R_\beta = R_{\alpha+\beta}$.

Reflection matrix

In line at angle θ to x-axis:

$$M_\theta = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}.$$

$M^2 = I$ — reflection is its own inverse.

Standard reflections

In x-axis: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

In y-axis: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

In $y = x$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Composite order

To apply T_1 first then T_2: write T_2T_1.

Read **right to left**.

Non-commutative: $T_2T_1 \neq T_1T_2$ in general.

Translations (homogeneous)

Linear maps fix origin. Translation needs 3×3 matrices:

$$T_{a,b} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}.$$

Point (x, y) as $(x, y, 1)^\top$.

Common pitfalls

- Applying transformations in wrong order.
- Confusing R_θ with M_θ.
- Forgetting homogeneous-coord conversion for translations.

SECTION C

Matrices — 2D Linear Transformations

Questions 1–2 · 16 marks

4. The transformation T in the plane consists of a reflection in the x-axis, followed by a translation in which the point (x, y) is transformed to the point $(x - 2, y + 1)$, followed by an anticlockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}. \quad [5]$$

(b) Show that T has no fixed points. [3]

8. The transformation T in the plane consists of a reflection in the line $y - x = 0$, followed by a translation in which the point (x, y) is transformed to the point $(x + 2, y - 1)$, followed by a reflection in the line $y + x = 0$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [5]

(b) Find the coordinates of the fixed point of T. [3]

END OF MATRICES — 2D LINEAR TRANSFORMATIONS PACK

Source: WJEC FP1 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 1 — Topic 5 (2.1.3)