Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER PURE MATHEMATICS A QUESTION PACK

2305-01 (Legacy FP1) · New spec Unit 1 Topic 4

REVISE
.wales

FURTHER MATHS — FP A · FURTHER ALGEBRA & SERIES

Roots of Polynomials & Summation of Finite Series

Every roots/series question from the legacy WJEC FP1 papers (June 2009 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 57 minutes

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (78 marks over 11 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every further algebra & series question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 4 (2.1.4).

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 09 Q1	6	
2	Jan 14 Q2	6	
3	Jun 13 Q1	6	
4	Jun 17 Q2	6	
5	Jun 10 Q6	7	
6	Jun 14 Q5	10	

Q	Source	Max	Mark
7	Jun 13 Q3	11	
8	Jun 10 Q4	8	
9	Jan 10 Q5	6	
10	Jun 15 Q5	5	
11	Jun 11 Q9	7	
Total		78	

Roots of Polynomials & Summation of Finite Series — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 1: Further Pure Mathematics A · Topic 2.1.4.

Roots of polynomials 2.1.4

- For $ax^2+bx+c=0$ with roots α, β: $\alpha+\beta = -b/a,\ \alpha\beta = c/a$.
- For $ax^3+bx^2+cx+d=0$ with roots α,β,γ: $\sum\alpha = -b/a, \sum\alpha\beta = c/a, \alpha\beta\gamma = -d/a$.
- For quartic $\alpha+\beta+\gamma+\delta = -b/a, \sum\alpha\beta = c/a, \sum\alpha\beta\gamma = -d/a, \alpha\beta\gamma\delta = e/a$.
- Equation with new roots: find symmetric functions of the new roots in terms of the old.

Symmetric sums 2.1.4

- $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$.
- $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$.
- For cubics: $\alpha^2+\beta^2+\gamma^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$.
- $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \dfrac{\sum\alpha\beta}{\alpha\beta\gamma}$.

Standard series 2.1.4

- $\sum_{r=1}^{n} r = \dfrac{n(n+1)}{2}$
- $\sum_{r=1}^{n} r^2 = \dfrac{n(n+1)(2n+1)}{6}$
- $\sum_{r=1}^{n} r^3 = \dfrac{n^2(n+1)^2}{4} = (\sum_{r=1}^{n} r)^2$
- Combine these (with $\sum$ linearity) to evaluate $\sum$ of any quadratic or cubic in r.

Method of differences 2.1.4

- For $\sum_{r=1}^{n}[f(r)-f(r+1)]$ the series *telescopes* — most terms cancel.
- Result: $f(1)-f(n+1)$.
- Often the summand decomposes via partial fractions: e.g. $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$.
- Useful when standard formulae don't apply directly.

Further Algebra & Series in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Vieta — quadratic

For $ax^2+bx+c=0$ with roots α,β:

$\alpha+\beta=-\dfrac{b}{a},\quad \alpha\beta=\dfrac{c}{a}$

Vieta — cubic

For $ax^3+bx^2+cx+d=0$ with roots α,β,γ:

$\sum\alpha=-\dfrac{b}{a}$

$\sum\alpha\beta=\dfrac{c}{a}$

$\alpha\beta\gamma=-\dfrac{d}{a}$

Vieta — quartic

For $ax^4+bx^3+cx^2+dx+e=0$ with roots $\alpha,\beta,\gamma,\delta$:

$\sum\alpha=-b/a$

$\sum\alpha\beta=c/a$

$\sum\alpha\beta\gamma=-d/a$

$\alpha\beta\gamma\delta=e/a$

Power sums

$\sum\alpha^2=(\sum\alpha)^2-2\sum\alpha\beta$

$\sum\alpha^3=(\sum\alpha)^3-3(\sum\alpha)(\sum\alpha\beta)+3\alpha\beta\gamma$ (for cubic).

$\sum\dfrac{1}{\alpha}=\dfrac{\sum\alpha\beta}{\alpha\beta\gamma}$

New equation with related roots

Given roots α,β,γ of one cubic, find equation with roots $\alpha+k$, α^2, $1/\alpha$, etc.

Substitute $y=$ new root expression, solve for x, replace in original equation.

E.g. for roots $\alpha+k$: substitute $x=y-k$ into original.

Standard sums

$\sum_{r=1}^{n}r=\frac{1}{2}n(n+1)$

$\sum_{r=1}^{n}r^2=\frac{1}{6}n(n+1)(2n+1)$

$\sum_{r=1}^{n}r^3=\frac{1}{4}n^2(n+1)^2$

Sigma linearity

$\sum(a\,u_r+b\,v_r)=a\sum u_r+b\sum v_r$

Use to break a polynomial summand into standard sums.

e.g. $\sum r(r+1)=\sum r^2+\sum r$.

Method of differences

If summand can be written as $f(r)-f(r+1)$, the series *telescopes*:

$\sum_{r=1}^{n}[f(r)-f(r+1)]=f(1)-f(n+1)$

Useful for $\sum\frac{1}{r(r+1)}$ via partial fractions.

Strategy

1. Write Vieta's relations from the given polynomial.
2. For series, check if a standard sum or method of differences applies.
3. Verify with $n=1$ (small case sanity check).
4. Factor your answer where possible.

SECTION A

Further Algebra & Series

Questions 1–11 · 78 marks

1. Given that

$$S_n = \sum_{r=1}^{n} r(r+1)^2 ,$$

find an expression for S_n in terms of n, giving your answer as a product of linear factors. [6]

2. Given that

$$S_n = 1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \cdots + n(n+1)^2,$$

obtain an expression for S_n, giving your answer as a product of linear factors. [6]

1. Given that

$$S_n = 1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2,$$

obtain an expression for S_n in the form $an^3 - bn$, where a, b are positive rational numbers. [6]

2. Consider the series

$$S_n = 1^2 + 4^2 + 7^2 + \ldots + (3n - 2)^2.$$

Obtain an expression for S_n, giving your answer in the form $an^3 + bn^2 + cn$, where a, b, c are rational numbers. [6]

6. *(a)* Express $\dfrac{1}{r(r+2)}$ in partial fractions. [3]

(b) Hence show that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{3}{4} - \frac{(2n+3)}{2(n+1)(n+2)} .$$ [4]

5. The roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0$$

are denoted by α, β, γ.

(a) Find the cubic equation whose roots are $\beta\gamma$, $\gamma\alpha$, $\alpha\beta$. [6]

(b) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = 0.$$

Deduce the number of real roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0,$$

justifying your answer. [4]

3. The roots of the cubic equation $x^3 - 2x^2 + 2x + 1 = 0$ are denoted by α, β, γ .

(a) Show that

$$\frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} + \frac{\alpha\beta}{\gamma} = -8.$$ [4]

(b) Find the cubic equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$. [7]

4. The roots of the quadratic equation

$$x^2 + 2x + 3 = 0$$

are denoted by α, β. Find the quadratic equation whose roots are

$$\alpha - \frac{1}{\beta^2},\ \beta - \frac{1}{\alpha^2}.$$

[8]

5. Given that the cubic equation $x^3 - qx + r = 0$ has two equal roots, show that

$$4q^3 = 27r^2.$$ [6]

5. The roots of the cubic equation

$$x^3 - 4x^2 - 8x + k = 0$$

are in geometric progression. Determine the value of k. [5]

9. The roots of the following cubic equation are in geometric progression.

$$x^3 + fx^2 + gx + h = 0$$

Show that $g^3 = f^3h$. [7]

END OF FURTHER ALGEBRA & SERIES PACK

Source: WJEC FP1 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 1 — Topic 4 (2.1.4)