Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER PURE MATHEMATICS A QUESTION PACK

2305-01 (Legacy FP1) · New spec Unit 1 Topic 3

REVISE
.wales

FURTHER MATHS — FP A · MATRICES

Matrix Algebra & Transformations

Every matrix question from the legacy WJEC FP1 papers (Jan 2007 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 14 minutes

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (89 marks over 11 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every matrices question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 3 (2.1.3).

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 17 Q1	7	
2	Jun 09 Q3	8	
3	Jan 14 Q7	7	
4	Jan 12 Q5	11	
5	Jan 13 Q4	10	
6	Jun 15 Q8	8	

Q	Source	Max	Mark
7	Jun 17 Q4	8	
8	Jan 11 Q8	8	
9	Jun 17 Q5	7	
10	Jan 07 Q5	7	
11	Jun 14 Q3	8	
	Total	89	

Matrix Algebra & Transformations — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 1: Further Pure Mathematics A · Topic 2.1.3.

Matrix algebra 2.1.3

- Add and subtract: element-wise.
- Scalar multiplication: $(kA)_{ij} = k\,a_{ij}$.
- Multiplication: $(AB)_{ij} = \sum_k a_{ik}b_{kj}$ — *not commutative*.
- Identity I, zero $\mathbf{0}$, transpose $A^\top$.

Determinants & inverses 2.1.3

- 2×2: $\det = ad - bc$. Inverse $\frac{1}{\det}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.
- 3×3: expand along any row/column with cofactors.
- Inverse via $A^{-1} = \frac{1}{\det A}\,\text{adj}\,A$.
- Singular iff $\det A = 0$.

Linear systems 2.1.3

- Solve $A\mathbf{x} = \mathbf{b}$ by computing $\mathbf{x} = A^{-1}\mathbf{b}$ if $\det A \neq 0$.
- Otherwise use row reduction to echelon form — outcomes: unique, infinite, or no solution.
- For consistency, the augmented column must not introduce a new pivot.

2D transformations 2.1.3

- Rotation by θ: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.
- Reflection in line at angle θ to x-axis: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$.
- Enlargement scale k: kI.
- Composite: apply *right-to-left* — matrix multiplication order matters.

Matrices in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Multiplication

$(AB)_{ij} = \sum_k a_{ik} b_{kj}$ — row i of A dot column j of B.

A is $m \times n$, B is $n \times p \Rightarrow AB$ is $m \times p$.

$AB \neq BA$ in general — matrix mult is *not* commutative.

Determinant 2×2 / 3×3

$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

3×3: expand along row 1:

$\det A = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ where M_{ij} is the 2×2 minor.

Inverse (2×2)

For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Singular iff $ad - bc = 0$.

Inverse (3×3)

$A^{-1} = \frac{1}{\det A}\operatorname{adj}(A)$.

$\operatorname{adj}(A)$ = transpose of the cofactor matrix.

Cofactor $C_{ij} = (-1)^{i+j}M_{ij}$ where M_{ij} is the 2×2 minor.

Solving systems

$A\mathbf{x} = \mathbf{b}$.

If $\det A \neq 0$: $\mathbf{x} = A^{-1}\mathbf{b}$ — unique solution.

If $\det A = 0$: use row reduction. Result is either *infinite* solutions (consistent) or *no* solutions (inconsistent).

Rotation matrix

Anticlockwise rotation by θ about the origin:

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Multiple rotations: $R_\alpha R_\beta = R_{\alpha+\beta}$.

Reflection matrix

Reflection in line through origin at angle θ to x-axis:

$$M_\theta = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

$M^2 = I$ — reflection is its own inverse.

Enlargement & composites

Enlargement scale factor k centred at origin: $kI = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$.

Composite transformations: **multiply right-to-left**. To apply T_1 then T_2: write T_2T_1.

Strategy

1. Compute $\det A$ first — tells you everything.
2. For transformations: identify pure rotation/reflection/enlargement, or composite.
3. For systems: row-reduce if $\det = 0$.
4. Watch the multiplication order in composites.

SECTION A

Matrices

Questions 1–11 · 89 marks

1. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}.$$

(a) Evaluate the determinant of $\mathbf{M}$. [2]

(b) (i) Find the adjugate matrix of $\mathbf{M}$.

(ii) Deduce the inverse matrix $\mathbf{M}^{-1}$. [3]

(c) Hence solve the system of equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \\ 17 \end{bmatrix}.$$

[2]

3. *(a)* Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix} .$$

[6]

(b) Hence solve the system of equations

$$\begin{aligned} x + 2y + 3z &= 13 \\ 2x + 3y + z &= 13 \\ 3x + 5y + 2z &= 22. \end{aligned}$$

[2]

7.

(a) Given that $\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$,

(i) find the adjugate matrix of $\mathbf{A}$,

(ii) find the inverse of $\mathbf{A}$. [5]

(b) **Hence** solve the equations

$$\begin{aligned} 2x + 3y + z &= 13, \\ x + 2y + 3z &= 13, \\ 2x + 3y + 4z &= 19. \end{aligned}$$

[2]

5. The matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} k & 1 & 6 \\ 1 & k & 4 \\ 0 & 1 & 1 \end{bmatrix}.$$

(a) Show that **A** is non-singular for all real values of k. [4]

(b) Given that $k = 3$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse matrix of **A**,

(iii) **hence** solve the equations

$$\begin{aligned} 3x + y + 6z &= 1, \\ x + 3y + 4z &= -1, \\ y + z &= -1. \end{aligned}$$

[7]

4. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & 3 & \lambda \\ 4 & 7 & 5 \end{bmatrix}.$$

(a) Find the values of λ for which **A** is singular. [5]

(b) Given that $\lambda = 1$,

(i) determine the adjugate matrix of **A**,
(ii) determine the inverse matrix $\mathbf{A}^{-1}$. [5]

8. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

(a) Show that

$$\mathbf{A}^2 = 2\mathbf{A} - \mathbf{I},$$

where $\mathbf{I}$ denotes the 2×2 identity matrix. [2]

(b) Using mathematical induction, prove that

$$\mathbf{A}^n = n\mathbf{A} - (n-1)\mathbf{I}$$

for all positive integers n. [6]

TURN OVER

4. The transformation T in the plane consists of a reflection in the x-axis, followed by a translation in which the point (x, y) is transformed to the point $(x - 2, y + 1)$, followed by an anticlockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [5]

(b) Show that T has no fixed points. [3]

8. The transformation T in the plane consists of a reflection in the line $y - x = 0$, followed by a translation in which the point (x, y) is transformed to the point $(x + 2, y - 1)$, followed by a reflection in the line $y + x = 0$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$

[5]

(b) Find the coordinates of the fixed point of T. [3]

5. Consider the following equations.

$$x + 3y - z = 1,$$
$$2x - y + 2z = 3,$$
$$3x - 5y + 5z = \lambda.$$

(a) Find the value of λ for which the equations are consistent. [4]

(b) For this value of λ, find the general solution of the equations. [3]

5. Consider the simultaneous equations

$$\begin{aligned} x + 2y - z &= 2 \\ 2x - y + z &= 3 \\ 4x - 7y + 5z &= 5. \end{aligned}$$

Given that these equations do not have a unique solution,

(a) show that the equations are consistent. [4]

(b) find the general solution to the equations. [3]

3. Consider the following equations.

$$\begin{aligned} x + 2y + 4z &= 3, \\ x - y + 2z &= 4, \\ 4x - y + 10z &= k. \end{aligned}$$

Given that the equations are consistent,

(a) find the value of k, [5]

(b) determine the general solution of the equations. [3]

END OF MATRICES PACK

Source: WJEC FP1 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 1 — Topic 3 (2.1.3)