Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER PURE MATHEMATICS A QUESTION PACK

2305-01 (Legacy FP1) · New spec Unit 1 Topic 2

REVISE
.wales

FURTHER MATHS — FP A · COMPLEX NUMBERS

Complex Numbers

Every basic complex-number question from the legacy WJEC FP1 papers (Jan 2006 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 54 minutes

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (76 marks over 10 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every complex numbers question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 2 (2.1.2).

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 10 Q2	7	
2	Jan 10 Q3	6	
3	Jun 14 Q4	6	
4	Jun 17 Q3	8	
5	Jan 13 Q3	8	
6	Jun 11 Q3	7	
7	Jan 10 Q1	7	
8	Jun 11 Q5	7	
9	Jan 14 Q3	10	
10	Jun 16 Q4	10	
	Total	**76**	

Complex Numbers — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 1: Further Pure Mathematics A · Topic 2.1.2.

Algebra over $\mathbb{C}$ 2.1.2

- Add, subtract, multiply, divide complex numbers in $x + iy$ form.
- Use $i^2 = -1$ throughout. Real and imaginary parts: $\mathrm{Re}(z), \mathrm{Im}(z)$.
- Complex conjugate $\bar{z} = x - iy$; uses include rationalising denominators.

Mod & argument 2.1.2

- $|z| = \sqrt{x^2 + y^2}$; argument $\arg z = \mathrm{atan2}(y, x)$.
- Conversion: $z = r(\cos\theta + i\sin\theta)$ — mod-arg form.
- Geometric meaning in the Argand diagram — modulus as length, argument as angle from positive real axis.

Polynomial roots 2.1.2

- Solve quadratics over $\mathbb{C}$: real coefficients $\Rightarrow$ roots in conjugate pairs.
- Cubics with one complex root: factor as $(z - \alpha)(z - \bar{\alpha})Q(z)$.
- Quartics with one complex root: split into two quadratic factors over $\mathbb{R}$.

Working scientifically general

- Always rationalise denominators (multiply by conjugate) before equating parts.
- Equating real and imaginary parts gives *two* real equations from one complex equation.
- Argand sketches help check arguments — especially signs.

Complex Numbers in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Algebra rules

$(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$

$(a+bi)(c+di) = (ac-bd) + (ad+bc)i$

$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{c^2+d^2}$ — multiply by conjugate.

Conjugate

$\overline{z} = x - iy$ for $z = x + iy$.

$z\bar{z} = x^2 + y^2 = |z|^2$ (always real, non-negative).

Conjugate-distributive: $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$, $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$.

Modulus & argument

$|z| = \sqrt{x^2 + y^2}$

$\arg z = \theta$ where $\tan\theta = y/x$ (quadrant matters!).

Conventional range: $-\pi < \arg z \le \pi$ (principal value).

Modulus-argument form

$z = r(\cos\theta + i\sin\theta)$ where $r = |z|, \theta = \arg z$.

Multiplication: moduli multiply, arguments add.

$|z_1 z_2| = |z_1||z_2|, \arg(z_1 z_2) = \arg z_1 + \arg z_2$.

Roots of real polynomials

If a polynomial has *real coefficients*, complex roots come in **conjugate pairs**.

So if $\alpha = a + bi$ is a root, so is $\bar{\alpha} = a - bi$.

Then $(z-\alpha)(z-\bar{\alpha}) = z^2 - 2az + (a^2 + b^2)$ — a real quadratic factor.

Quartic factorisation

Given $z^4 + \ldots = 0$ with one complex root α:

The quartic has factor $(z^2 - 2\,\mathrm{Re}(\alpha)\,z + |\alpha|^2)$.

Divide quartic by this factor — you get a real quadratic with the other two roots.

Argand sanity check

After computing $|z|$ and $\arg z$, plot z on the Argand plane mentally.

z should lie in the quadrant matching $\mathrm{sign}(x), \mathrm{sign}(y)$.

Catches sign errors in arg.

Common pitfalls

- Computing arg as $\arctan(y/x)$ without checking quadrant — can be off by π.
- Forgetting the "$\pm$ pair" rule when finding other roots of a real polynomial.
- Mixing up $\bar{z}$ and $-z$.

Strategy

1. Rationalise denominators first.
2. Equate real and imaginary parts — two real equations.
3. For roots: use the conjugate, then factor.
4. For mod/arg: get the right quadrant.

SECTION A

Complex Numbers

Questions 1–10 · 76 marks

2. The complex number $z = 2 - \text{i}$. The complex conjugate of z is denoted by $\overline{z}$.
Find the modulus and argument of the complex number

$$z - \frac{5\overline{z}}{z}\ .$$ [7]

3. The complex number z is given by

$$z = \frac{1 + 8i}{1 - 2i}.$$

(a) Express z in the form $x + iy$. [3]

(b) Find the modulus and argument of z. [3]

4. The complex number z is given by

$$z = \frac{1+2\mathrm{i}}{1-\mathrm{i}}.$$

Find the modulus and the argument of z. [6]

3. The complex number z is given by $z = \dfrac{(1+2\text{i})(-3+\text{i})}{(1+3\text{i})}$.

Determine the modulus and the argument of z. [8]

3. The complex number z and its complex conjugate $\bar{z}$ satisfy the equation

$$iz + 2\bar{z} = \frac{4 + 6i}{1 + i}.$$

(a) Determine z in the form $x + iy$. [6]

(b) Find the modulus and the argument of z. [2]

3. Given that the complex number z and its complex conjugate $\bar{z}$ satisfy the equation

$$2\bar{z} + \mathrm{i}z = (1 + 2\mathrm{i})(2 - 3\mathrm{i}),$$

find z in the form $x + \mathrm{i}y$. [7]

1. *(a)* Show that 1 + 2i is a root of the equation $x^3 + x + 10 = 0$. [3]

(b) Determine the other two roots of the equation. [4]

5. Consider the polynomial equation

$$x^4 - 2x^3 - 2x^2 + 6x + 5 = 0.$$

Given that one of the roots of this equation is $2 + i$, determine all the other roots of the equation. [7]

3. *(a)* Express $(1 + 2\mathrm{i})^4$ in the form $x + \mathrm{i}y$, where x, y are real. [2]

(b) (i) Hence show that $1 + 2\mathrm{i}$ is a root of the quartic equation $x^4 + 12x - 5 = 0$.

(ii) Determine the other three roots of the equation. [8]

4. The complex numbers z_1, z_2 are given by

$$z_1 = -\sqrt{3} + \mathrm{i}; \;\; z_2 = 1 + \mathrm{i}.$$

(a) Determine the modulus and the argument of each of z_1, z_2, giving **exact** values of the moduli and giving the arguments in terms of π. [4]

(b) The complex number w is given by

$$w = \frac{z_1^{\,2}}{z_2}\,.$$

Using your results from *(a)*, or otherwise, determine w in the form $a + \mathrm{i}b$, giving a, b correct to two decimal places. [6]

END OF COMPLEX NUMBERS PACK

Source: WJEC FP1 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 1 — Topic 2 (2.1.2)