Name	Date started	Target end date

GCE AS / A LEVEL — FURTHER PURE MATHEMATICS A QUESTION PACK

2305-01 (Legacy FP1) · New spec Unit 1 Topic 1

REVISE
.wales

FURTHER MATHS — FP A · PROOF (INDUCTION)

Proof by Mathematical Induction

Every induction question from the legacy WJEC FP1 papers (June 2005 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 27 minutes

*Derived from the legacy FP1 paper's pace of **~1.5 min/mark** (58 marks over 8 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every proof (induction) question from the legacy WJEC FP1 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 1 (2.1.1).

Questions are ordered roughly by difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jan 07 Q4	7	
2	Jan 08 Q7	8	
3	Jan 09 Q6	8	
4	Jun 09 Q5	8	

Q	Source	Max	Mark
5	Jan 13 Q6	7	
6	Jun 14 Q8	7	
7	Jun 17 Q6	7	
8	Jun 16 Q7	6	
	Total	**58**	

Proof by Mathematical Induction — what the new spec asks

WJEC GCE AS / A Level Further Mathematics (from 2017) · Unit 1: Further Pure Mathematics A · Topic 2.1.1.

Mathematical induction 2.1.1

- State the principle: prove $P(1)$, assume $P(k)$, deduce $P(k+1)$, conclude $\forall n \in \mathbb{Z}^+$.
- Apply to sums of finite series: $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$, etc.
- Apply to divisibility statements: e.g. $5^{2n} - 1$ divisible by 24.
- Apply to powers of matrices: prove a closed form for A^n.

Knowledge assumed 2.1.1

- Sigma notation ($\sum$), index manipulation, factorisation.
- Familiarity with sequences and series from AS Mathematics.
- Algebraic identities, especially difference of two squares and factor theorem.

Common slip-ups general

- Forgetting the base case — *P(1)* must be verified explicitly.
- Confusing "assume $P(k)$" with "assume $P(n)$ for all n" (circular reasoning).
- Not stating the conclusion explicitly — mark schemes require it.

Working scientifically general

- Lay out clearly: **(i)** Base, **(ii)** Inductive Hypothesis, **(iii)** Inductive Step, **(iv)** Conclusion.
- In the inductive step, manipulate to *reach* the $P(k+1)$ form — show every line.
- Use $P(k+1) - P(k)$ comparisons to simplify series-sum proofs.

Proof (Induction) in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Induction template

Base case. Verify $P(1)$ (or smallest valid n).

Inductive hypothesis. Assume $P(k)$ holds for some $k \geq 1$.

Inductive step. Show $P(k+1)$ follows from $P(k)$ by algebra.

Conclusion. By induction $P(n)$ holds $\forall n \in \mathbb{Z}^+$.

Sum of series

To prove $\sum_{r=1}^{n} u_r = S(n)$:

$$\sum_{r=1}^{k+1} u_r = \underbrace{\sum_{r=1}^{k} u_r}_{=S(k)\text{ by IH}} + u_{k+1}$$

Show this equals $S(k+1)$ by algebraic manipulation.

Divisibility

To prove "$f(n)$ divisible by d":

Write $f(k+1) = f(k) + g(k)$ where $f(k) \equiv 0 \pmod{d}$ by IH.

Show $g(k) \equiv 0 \pmod{d}$ separately.

e.g. $5^{2n} - 1$ *div by 24: write* $5^{2(k+1)} - 1 = 25(5^{2k} - 1) + 24$.

Powers of matrices

To prove $A^n = M(n)$ for a closed-form matrix M:

Verify base: compute A^1, check matches $M(1)$.

Step: $A^{k+1} = A \cdot A^k = A \cdot M(k)$. Multiply and show $= M(k+1)$.

Recursive sequences

Given $u_{n+1} = f(u_n)$ and conjecture $u_n = g(n)$:

Base: u_1 matches $g(1)$.

Step: $u_{k+1} = f(u_k) = f(g(k))$; simplify to $g(k+1)$.

Standard sums (memorise)

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Common pitfalls

- Skipping the base case — loses marks even if step is perfect.
- Algebra error in the step — aim to reach the exact form of $S(k+1)$ in factored form.
- Forgetting to write the conclusion sentence.

Marking style

Mark schemes give credit for: (a) base case, (b) clear IH statement, (c) the algebra of the step shown line-by-line, (d) the explicit conclusion.

Skipping the IH statement loses marks even with correct algebra.

Strategy

1. Write base case explicitly.
2. State IH using k exactly.
3. Manipulate $u_{k+1} + S(k)$ or similar to reach $S(k+1)$.
4. Conclude with the standard sentence.

SECTION A

Proof (Mathematical Induction)

Questions 1–8 · 58 marks

4. Use mathematical induction to show that $6^n + 4$ is divisible by 5 for all positive integers n. [7]

7. Use mathematical induction to show that

$$\sum_{r=1}^{n} r \times 2^r = 2^{n+1}(n-1) + 2$$

for all positive integers n. [8]

6. Use mathematical induction to show that

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n & 2n^2 \\ 0 & 1 & 2n \\ 0 & 0 & 1 \end{bmatrix}$$

for all positive integers n. [8]

5. Use mathematical induction to prove that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1} \quad .$$

for all positive integers n. [8]

6. Use mathematical induction to prove that

$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

for all positive integers n. [7]

8. Using mathematical induction, prove that

$$\sum_{r=1}^{n}\left(r \times 2^{r-1}\right) = 1 + 2^{n}(n-1),$$

for all positive integers n. [7]

6. Use mathematical induction to prove that $9^n - 1$ is divisible by 8 for all positive integers n. [7]

7. The sequence $x_1, x_2, x_3, \ldots$ is generated by the relationship

$$x_{n+1} = 2x_n - n + 1 \quad \text{where} \quad x_1 = 3.$$

Use mathematical induction to prove that

$$x_n = 2^n + n$$

for all positive integers n. [6]

END OF PROOF (INDUCTION) PACK

Source: WJEC FP1 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec AS Unit 1 — Topic 1 (2.1.1)